PAUL SCHERRER INSTITUT

Wir schaffen Wissen – heute für morgen

Muon Spin Rotation (µSR) technique and its applications in magnetism and superconductivity

Zurab Guguchia

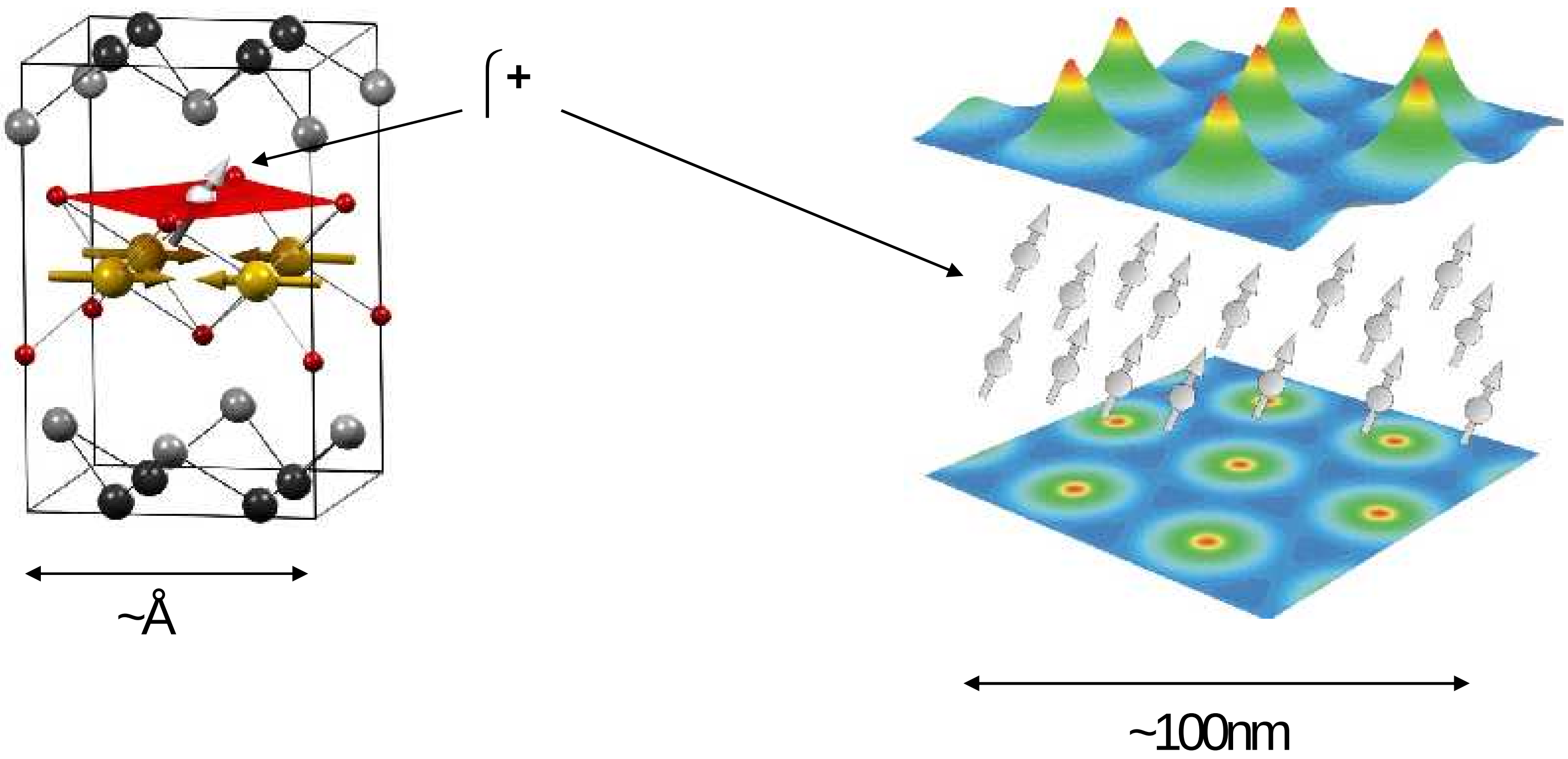
Laboratory for Muon Spin Spectroscopy

Paul Scherrer Institut

Muon is a Local Magnetic Probe

Muon probes the local magnetism from within the unit cell

Muon probes the local magnetic response of a superconductor (Meissner screening or flux line lattice)



Outline

1. **Muon Properties**
   - Pion decay
   - Muon decay
   - Parity violation
   - Muon spin precession

2. **Muon Spin Rotation / Relaxation (μSR)**
   - Facilities around the world
   - Muon production at PSI
   - μSR instruments at PSI
   - μSR principle
   - Muon thermalization / Muon stopping sites / Muon stopping ranges
   - Measurement geometries

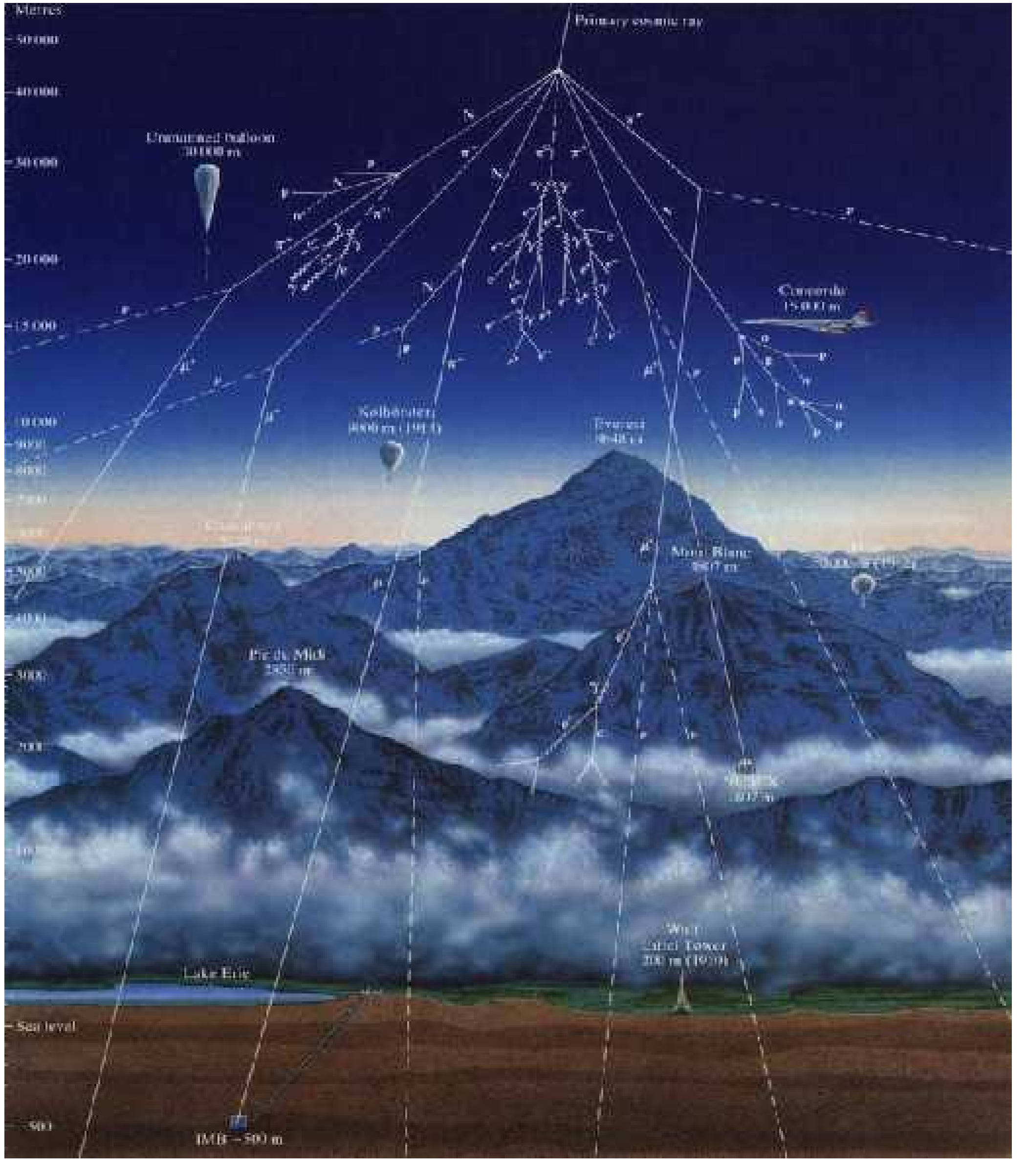
3. **Muon Spin Rotation / Relaxation on Magnetic Materials**
   - Different static depolarization functions and examples
   - Magnetic phase separation / coexistence of different magnetic phases
   - Magnetic fluctuations

4. **Muon Spin Rotation on Superconducting Materials**
   - Using low energy μSR to study the Meissner state of superconductors
   - Using bulk μSR to study the Vortex state of superconductors
   - Superfluid density and the symmetry of the superconducting gap
   - Magnetic and superconducting phase diagrams of Fe-based materials

5. **Summary**

Muon Properties

What is a Muon? Cosmics

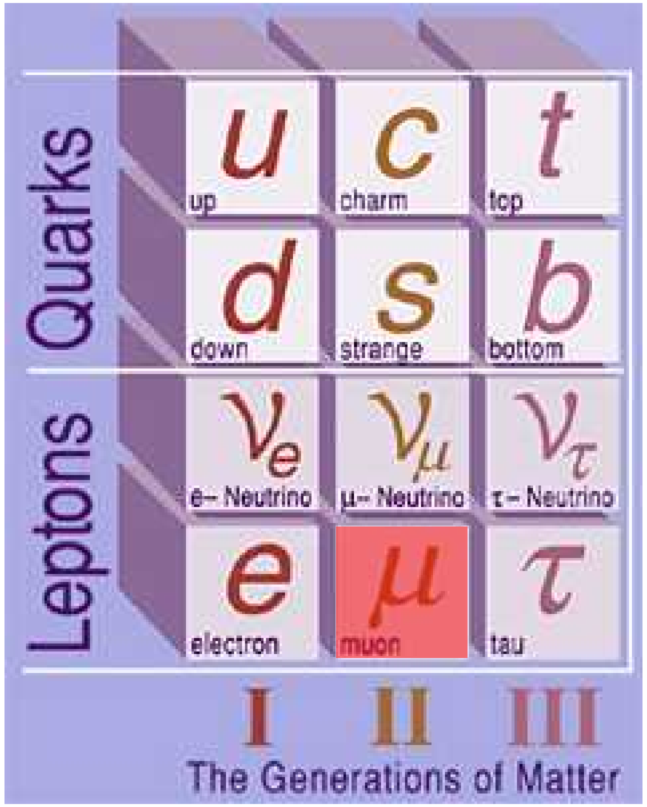


Muon Flux at sea level:
~ 1 Muon/Minute/cm^2

Mean Energy:
~ 2 GeV

Elementary particle/antiparticle:

mass:	< 200 x electron mass (105.6MeV/c^2) < 1/9 x proton mass
charge:	+ e, oder - e
spin:	1/2
magnetic moment :	3.18 x μ_p (8.9 x μ_N), g H 2.00
gyromagnetic ratio:	85.145 kHz/G
unstable particle:	**mean lifetime: 2.2 μs** **N(t) = N(0)exp(-t/│ ⌈)**



Muon production and polarised beams

Pions as intermediate particles

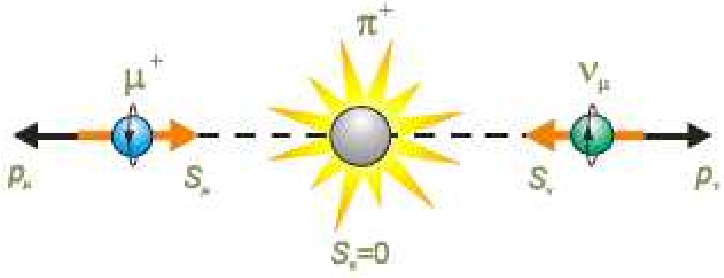
Protons of 600 to 800 MeV kinetic energy interact with protons or neutrons of the nuclei of a light element target to produce pions.

$$p + p \rightarrow \pi^{+} + p + n$$

Pions are unstable (lifetime 26 ns).
They decay into muons (and neutrinos):

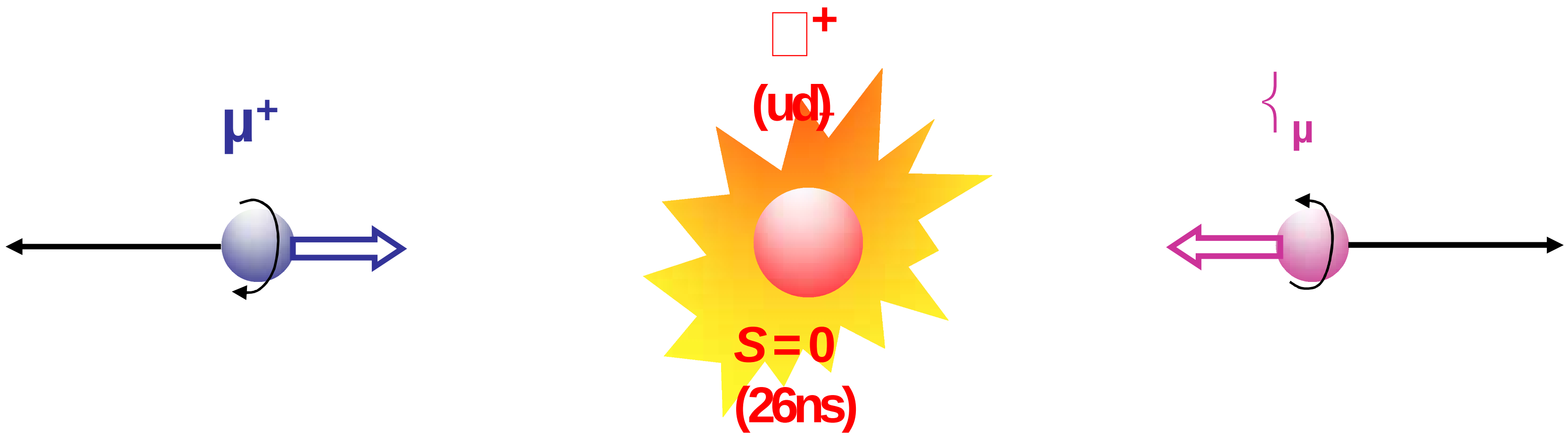
$$\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$$

The muon beam is 100 polarised with S_μ antiparallel to P_μ.



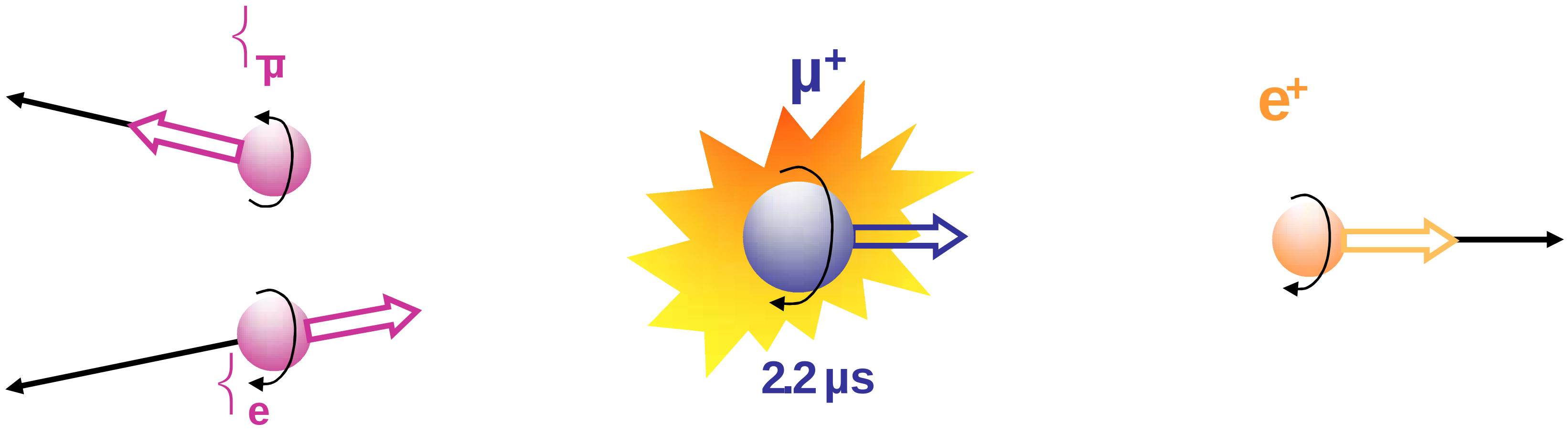
Momentum: P_μ=29.79 MeV/c. Kinetic energy: E_μ=4.12 MeV.

Muon as Result of Pion Decay



Two-body decay	▶	muon has always the energy 4.1 MeV in the reference frame of the pion (assuming $m_\nu = 0$)
Spin pion = 0	▶	Muon has a spin 1/2 and is 100% polarized (as only left-handed neutrinos are produced)

Muon Decay



Three-body decay	▶	Distribution of positrons energies
Weak-decay of muon	▶	Parity-violation leading to positrons emitted anisotropically

Measuring P(t): Muon Decay $\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$

- Muon decay (life time 2.2. μs) violates parity conservation

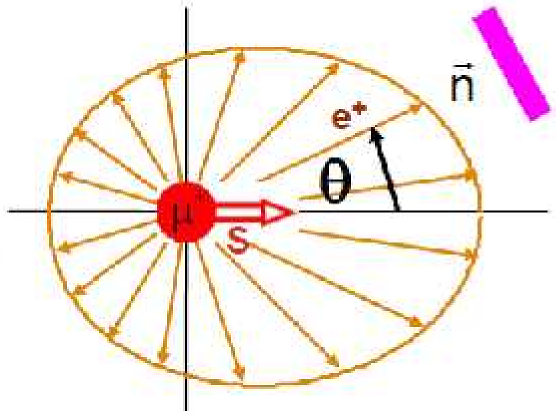
→ asymmetric decay

- Positrons preferentially emitted along muon spin (along polarization vector of muon ensemble)

$$\frac{dN_{e^+}(\theta)}{d\Omega} \propto \left(1+\frac{1}{3}P\cos\theta\right) = \left(1+\frac{1}{3}\vec{P}\cdot\vec{n}\right)$$

$\vec{n}$: direction of observation (detector position)

For P = 1:



$$\frac{dN_{e^+}(\theta)}{d\Omega} \propto \left(1+\frac{1}{3}P\cos\theta\right)$$

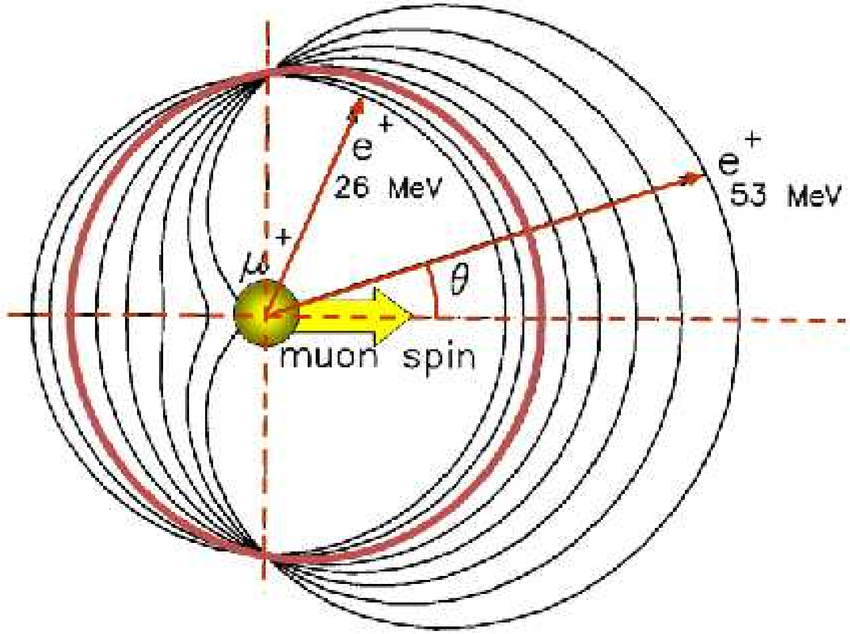
- Measuring positrons allows to observe time evolution of the polarization P(t) of the muon ensemble
- Positron intensity as a function of time after implantation:

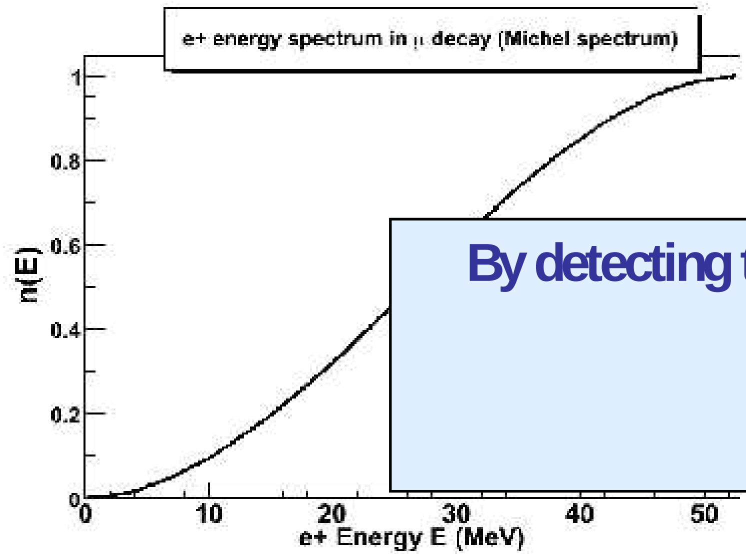
$$N_{e^+}(t) = N_0\left[1+A_0P(t)\right] e^{-\frac{t}{\tau_\mu}} \qquad P(t) = \vec{P}(t)\cdot\vec{n}$$

- A_0: Maximum observable asymmetry
  theoretically: A_0=1/3
  practically it depends on setup (average over solid angle, absorption in materials): A_0 = 0.25 - 0.30
- $A_0P(t)$ is called asymmetry: A(t)

θ : angle between spin (polarization) and positron direction

Anisotropic Muon Decay





Angular distribution of positrons from the parity violating muon decay:

$$W(E,\theta) = 1 + a(E)\cos(\theta)$$

The asymmetry parameter *a* = 1/3 when all positron energies E are sampled with equal probability.

Positrons preferentially emitted along direction of muon spin at decay time

By detecting the spatial positron emission as a function of time

▼

time evolution of muon spin !!!

Muon Spin Precession – Larmor frequency

$$|\psi\rangle = |\uparrow\rangle$$

$$\chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \Uparrow$$

$\mathbf{B}_\mu$

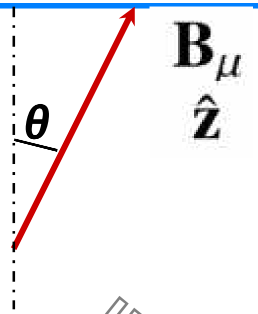
$\hat{\mathbf{z}}$

- Field axis: quantization axis
- Spin-state eigenvalue of S_z
- Stationary state

Muon Spin Precession – Larmor frequency

$\cancel{|\psi\rangle = |\uparrow\rangle}$

$\cancel{\chi = \begin{pmatrix}1\\0\end{pmatrix}}$ $\Uparrow$



- Field axis: quantization axis
- ~~Spin-state eigenvalue of S_z~~
- ~~Stationary state~~

Quantum Mechanics:

$\mathbf{B}_\mu$ quantization axis

χ in the new base $\Rightarrow \chi = \begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}$

Calculation of $\chi(t)$

with time dependent Schrödinger Eq.

Calculation expectation values of S_α

$\langle S_\alpha\rangle = \chi^\dagger S_\alpha \chi$

Project back to the ref. frame of the lab.

Classically:

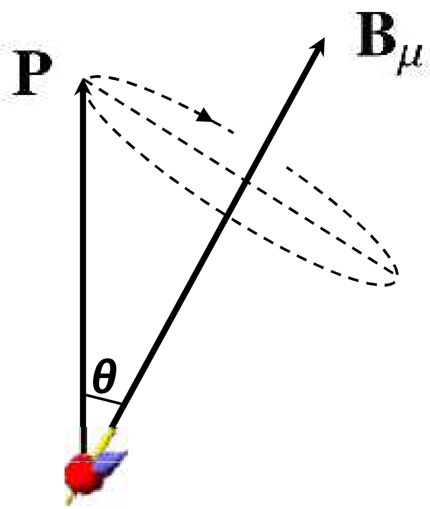
Torque: $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}_\mu = \gamma_\mu \mathbf{S}\times\mathbf{B}_\mu$

Euler's Eq.: $\dfrac{d\mathbf{S}}{dt} = \boldsymbol{\tau}$

$\Rightarrow \dfrac{d\mathbf{S}}{dt} = \gamma_\mu \mathbf{S}\times\mathbf{B}_\mu$

Muon Spin Precession – Larmor frequency

$$\mathbf{P} = \frac{\langle \mathbf{S} \rangle}{\frac{\hbar}{2}}$$



Classically:

Torque: $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}_\mu = \gamma_\mu \mathbf{S} \times \mathbf{B}_\mu$

Euler's Eq.: $\dfrac{d\mathbf{S}}{dt} = \boldsymbol{\tau}$

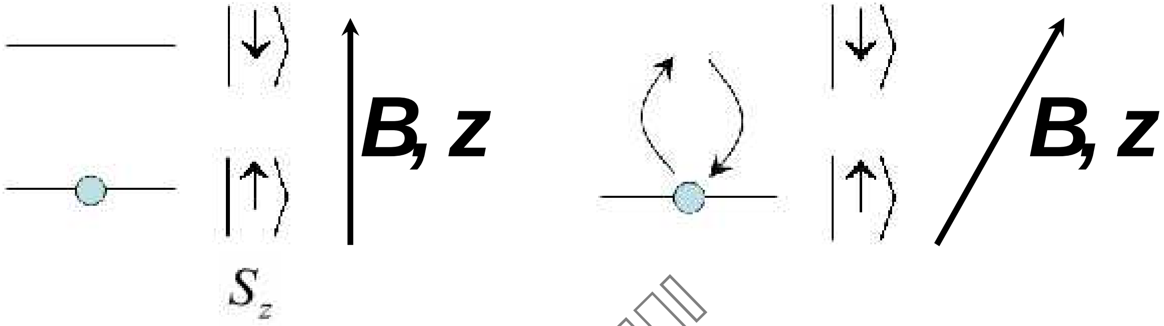
$\Rightarrow \dfrac{d\mathbf{S}}{dt} = \gamma_\mu \mathbf{S} \times \mathbf{B}_\mu$

Larmor precessions with angular velocity: $\omega_\mathrm{L} = \gamma_\mu B_\mu$

with $\gamma_\mu = \dfrac{e}{2m_\mu} g_\mu = 8.51615 \times 10^8$ rad/sT

Frequency: $\dfrac{\gamma_\mu}{2\pi} = 135.539$ MHz/T

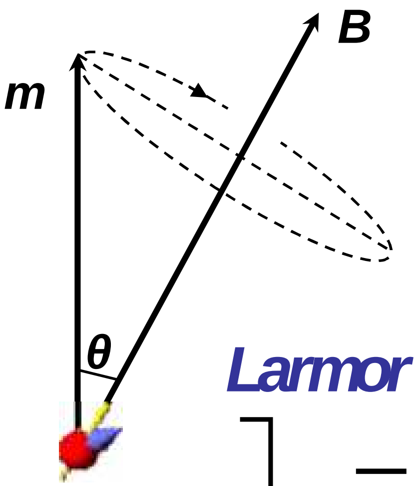
Muon Spin Precession - Bloch Equation



Rabi Oscillations

$\langle S_x \rangle$ and $\langle S_y \rangle$ precess with the Larmor frequency ω_L

Ehrenfest Theorem



Bloch Equation

$$\frac{d\vec{m}}{dt} = \gamma\, \vec{m} \cdot \vec{B}$$

Larmor frequency

$$\omega_L = \gamma \left| \vec{B} \right|$$

Summary - Muon Properties

The three key properties making μSR possible:

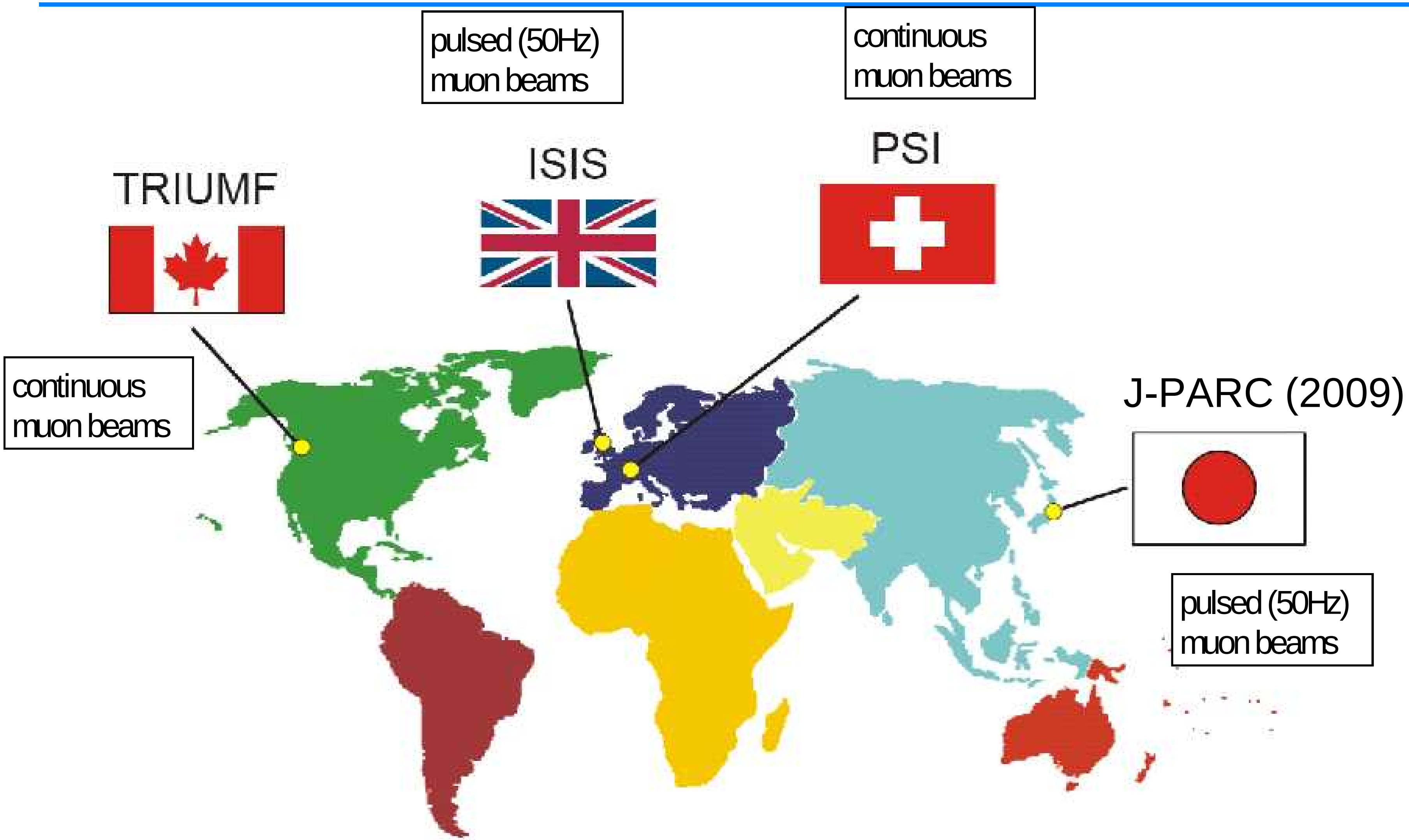
1. The muon is 100% spin polarized.

2. The decay positron is preferentially emmitted along the muon spin direction.

3. The muon spin precesses in a magnetic field.

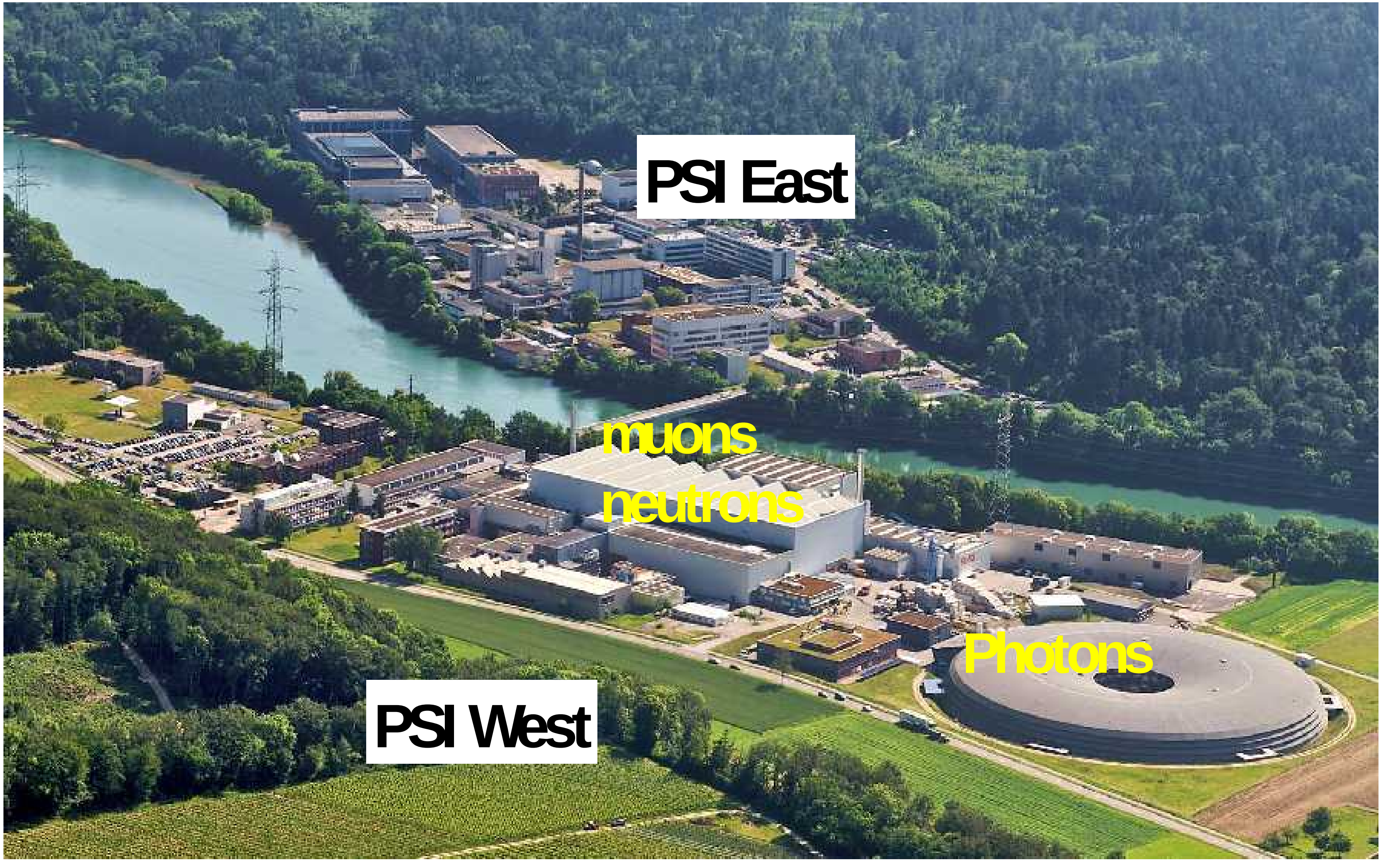
Muon Spin Rotation / Relaxation

μSR Facilities around the World



Facilities under study in South Corea, China, US

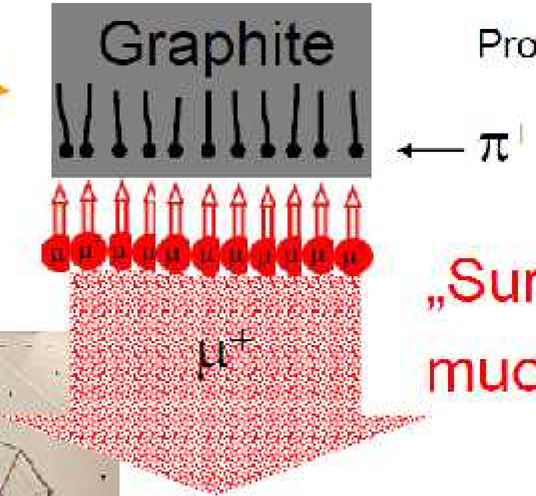
From "μSR brochure" by J.E Sonier, Simon-Fraser-Univ., Canada, 2002. http://musr.org/intro/musr/muSRBrochure.pdf


PSI East
muons
neutrons
Photons
PSI West

Generation of polarized muons (μ^+)

2.2 mA $\cong$ 1.4 10^{16} Protons/sec

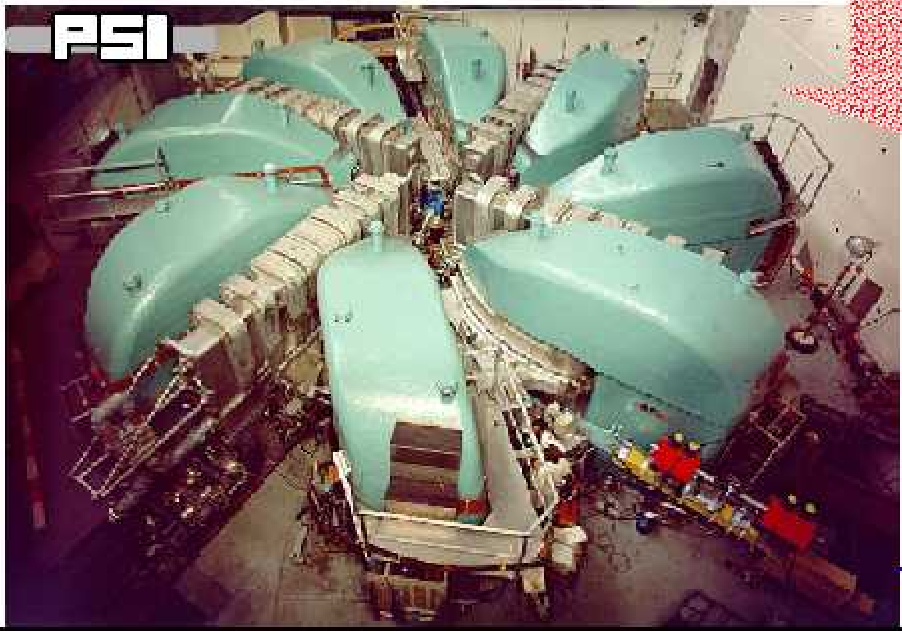
with 600 MeV



Production Target

$p + C \rightarrow \pi^+ \pi^- p\ n \ldots$

„Surface“ muons



$\sim 10^7 - 10^8\ \mu^+$/sec

100 % pol.

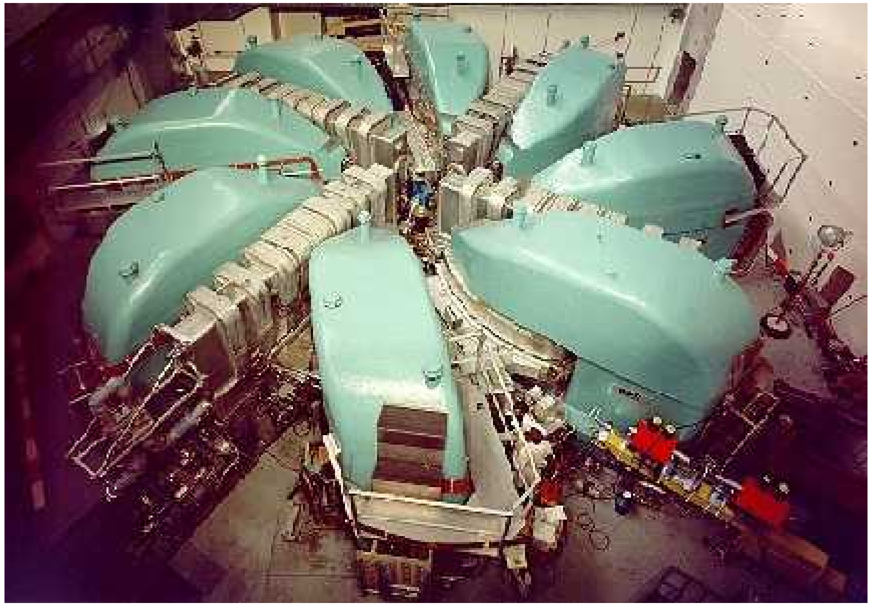
~ 4 MeV

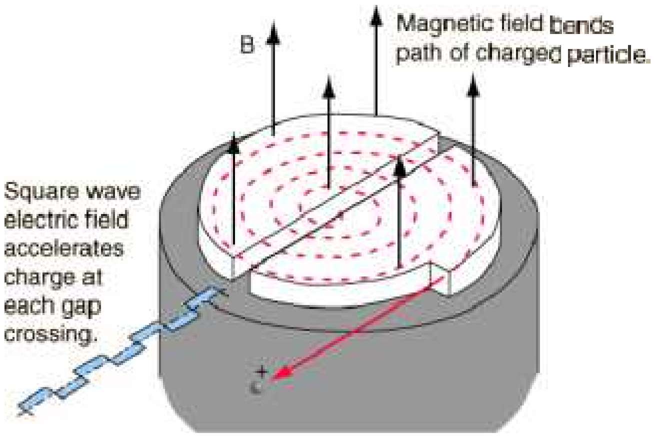
generally used for “bulk” condensed matter studies

For thin film studies: eV-30 keV

Muon Production at PSI

600 MeV Proton Cyclotron at PSI:

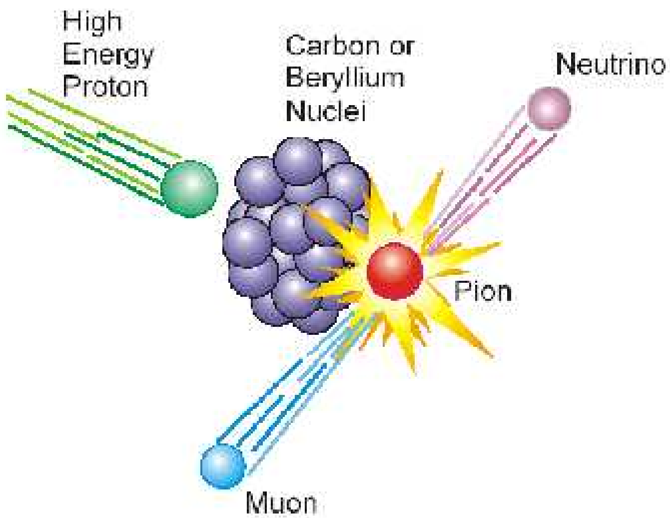




Muons from Pion Decay:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$



Muon kinetic energy: 4 MeV

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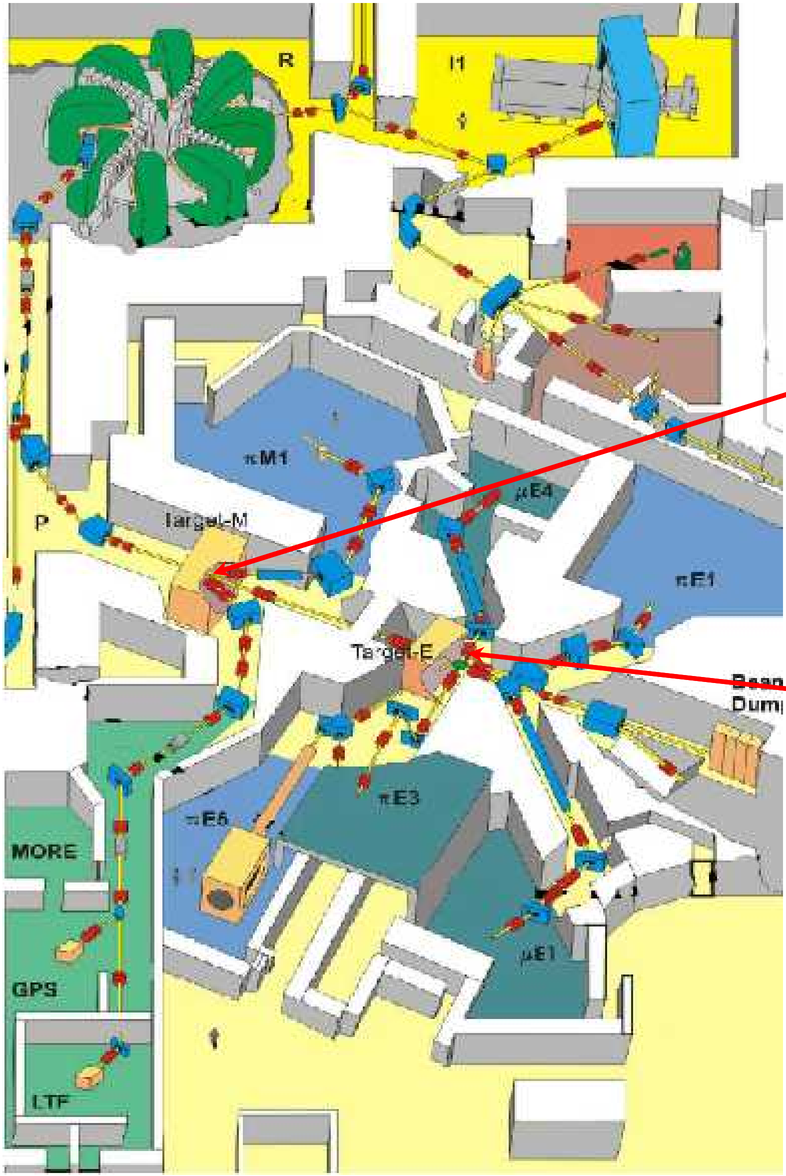
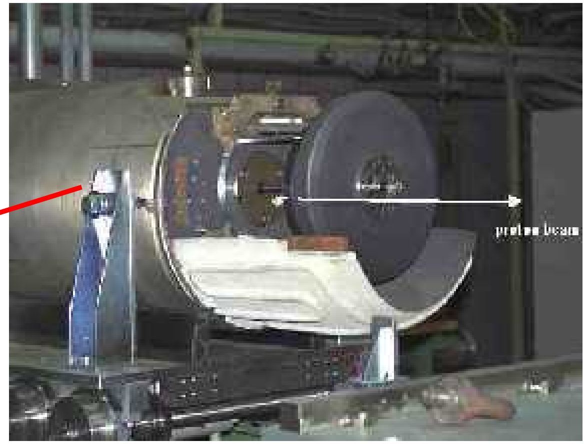
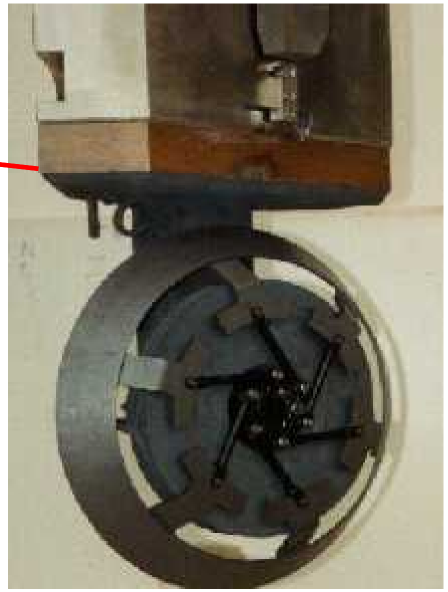
PSI Experimental Hall



Target M
Thin graphite (5 mm)

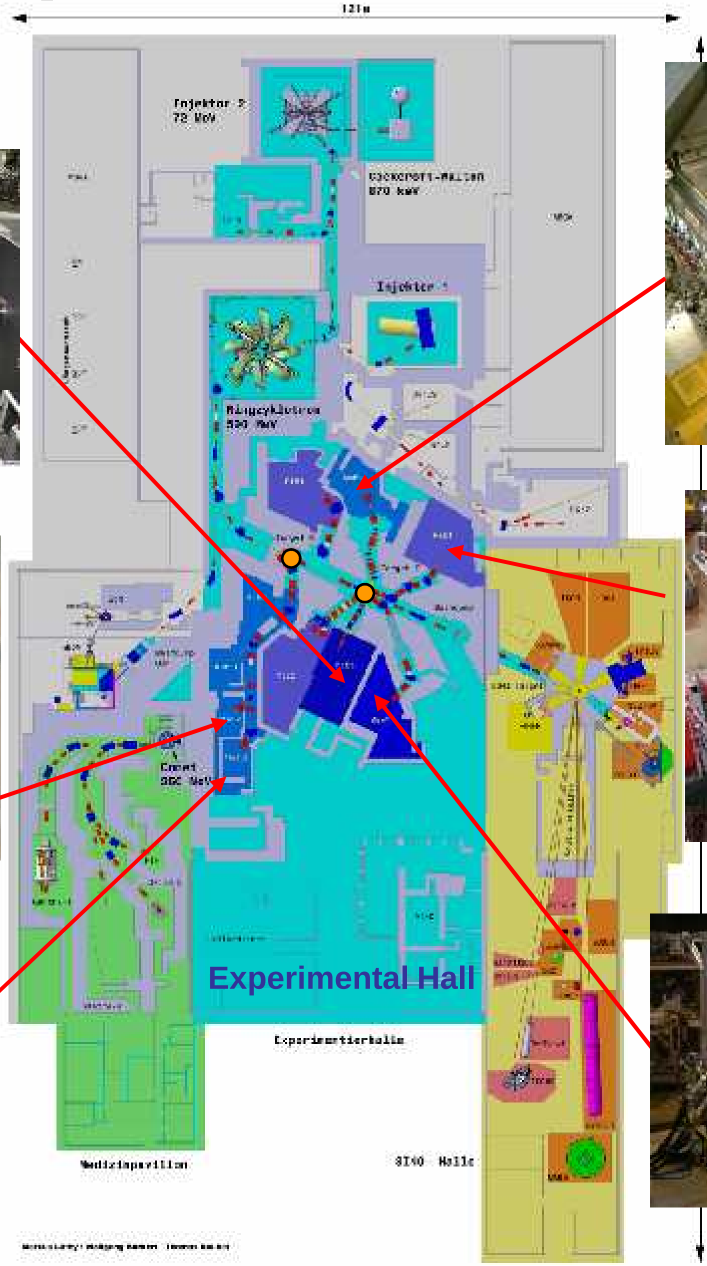




Target E
4cm graphite

"necessary component to expand proton beam before SINQ"

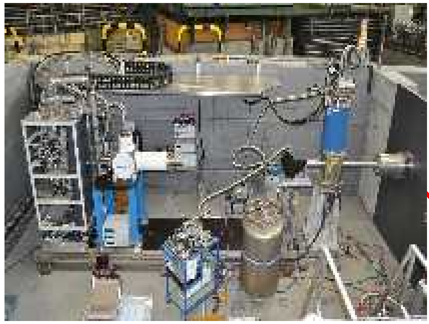
SµS – The Swiss Muon Source



High Field µSR

Muon energy:
4.2 MeV (μ^+)

9.5T, 20mK



GPS

General Purpose Surface Muon Instrument
Muon energy: **4.2 MeV** (μ^+)

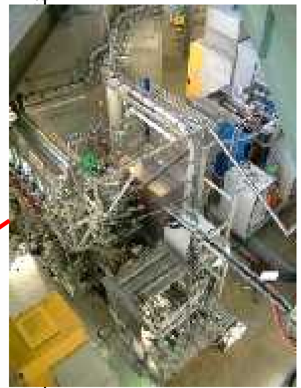
0.6T, 1.8K

Shared Beam Surface Muon Facility (Muon On REquest)

LTF

Low Temperature Facility
Muon energy: **4.2 MeV** (μ^+)

3T, 20mK- 4K



LEM

Low-energy muon beam and instrument, tunable energy (**0.5-30 keV**, μ^+), *thin-film, near-surface and multi-layer studies (1-300 nm)*

0.3T, 2.9K

DOLLY

General Purpose Surface Muon Instrument
Muon energy: **4.2 MeV** (μ^+)

0.5T, 300mK

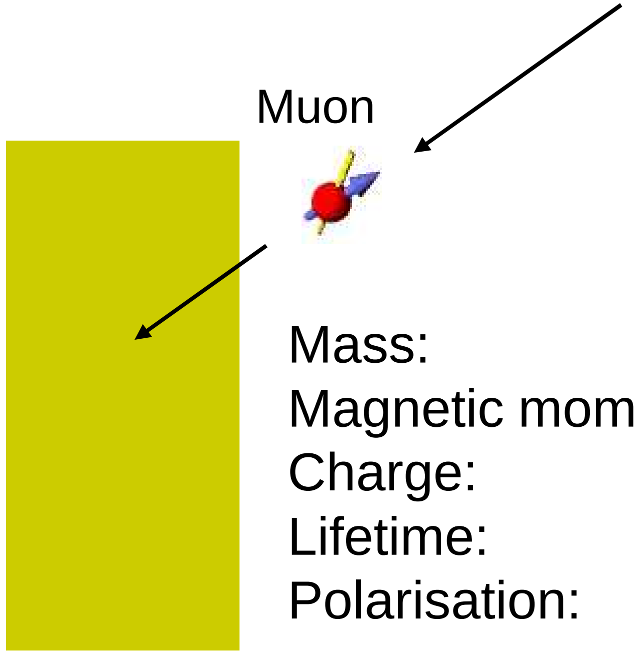
GPD

General Purpose Decay Channel Instrument
Muon energy: **5 - 60 MeV** (μ^+ or μ^-)

0.5T, 300mK
2.8GPa

Principle of a μSR Experiment

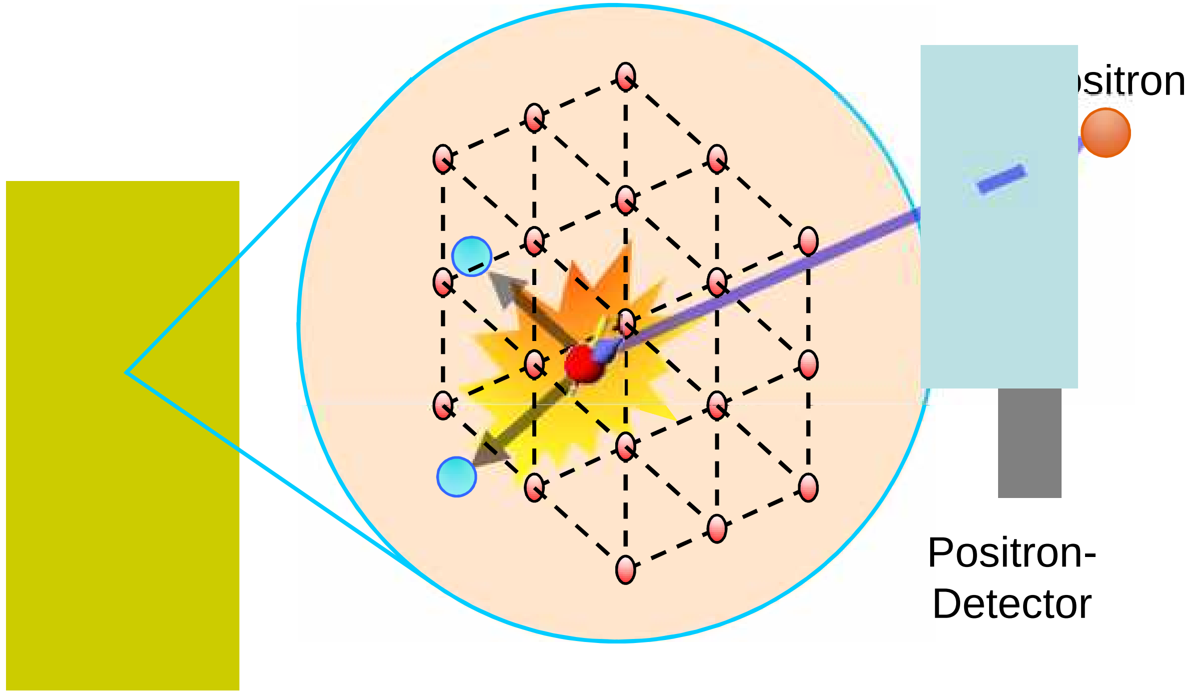
Implantation of muons into the sample



Mass:	$m_\mu \approx 207\, m_e \approx 1/9\, m_p$
Magnetic moment:	$\mu_\mu \approx 3\, \mu_p$
Charge:	+e
Lifetime:	$t_\mu \approx 2.2\ \mu s$
Polarisation:	100 %

Principle of a μSR Experiment

Detection of the decay positron



μSR

2 μs

Static field distribution	Fluctuation rates
Sensitivity: $10^{-3} - 10^{-4}\ \mu_B$	$(10^5 - 10^9$ Hz)

Muons stopping in matter:

4.1-MeV μ^+, v ~ 0.27· c

↓

ionization of atoms, 10^5-10^6 excess e^-

↓

2 – 3 keV, v ~ 0.007· c

↓

Muonium formation (μ^+e^-), successive e^- capture and loss

↓

100 eV, v ~ 0.0013· c

↓

mainly elastic collisions, stop at intersitial site

- Total stopping time in condensed matter: < 0.1 ns
- Only electrostatic processes
  ➨ no loss of polarization
- Penetration: ~0.1 mm

μSR – Muons Stopping in Matter

Muon Production

Surface Muons
Energy: 4.1 MeV
$v \approx 0.27\,c$

Sample

Ionization of atoms

$10^5 - 10^6$ excess e^-
2-3 keV
$v \approx 0.007\,c$

Muonium formation (μ^+e^-)
charge cycling

100 eV
$v \approx 0.0013\,c$

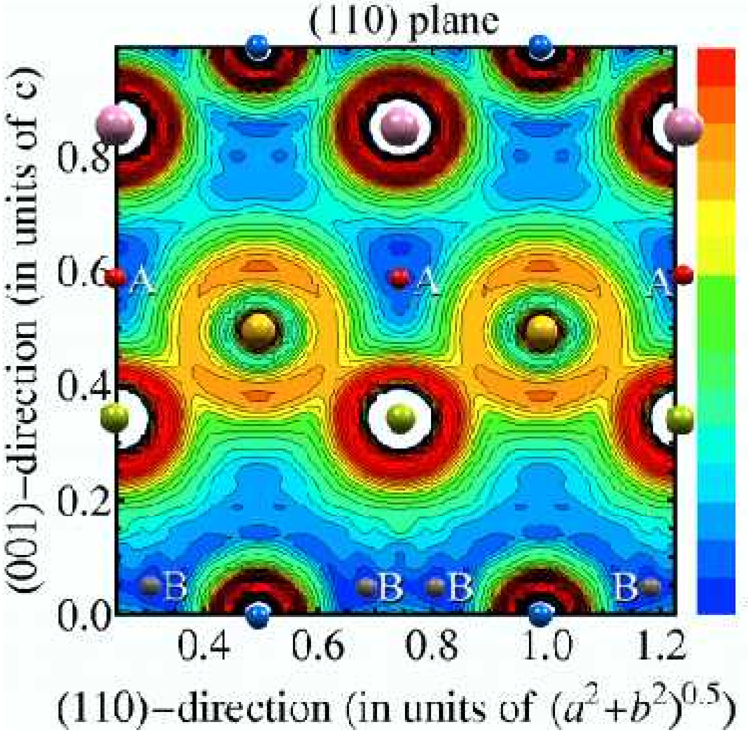
mainly elastic collisions
(dissociation, $\mu^+e^- \rightarrow \mu^+$)
stop at interstitial site

thermalized μ^+
stopping at an interstitial site

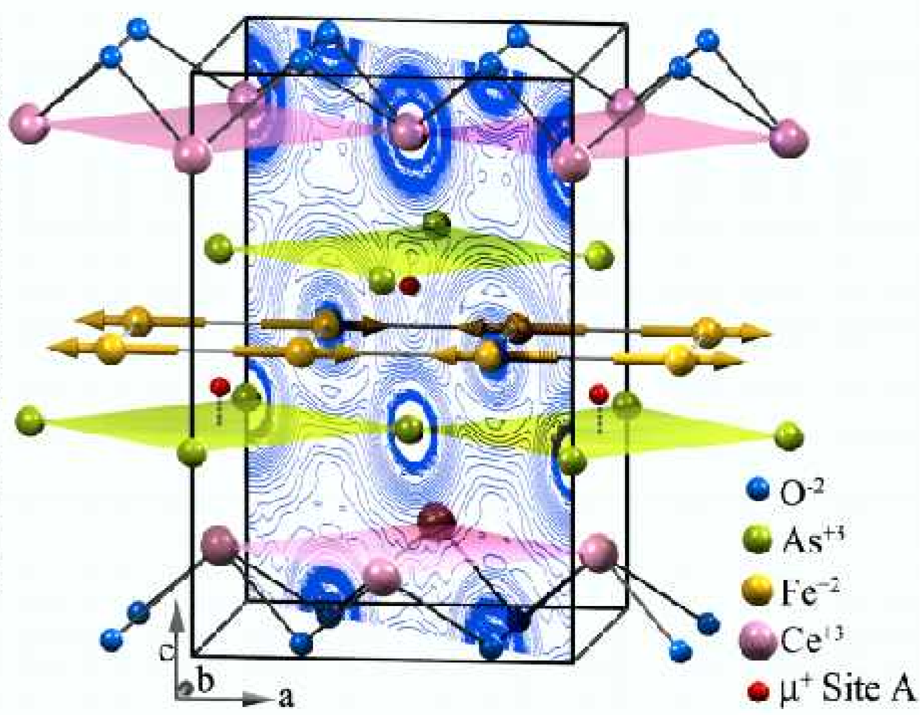
- Total stopping time in matter < 0.1 ns
- Only electrostatic processes
  → no loss of polarization!
- Penetration ≈ 0.1 mm

Interstitial Muon Stopping Sites

Electrostatic potential map:



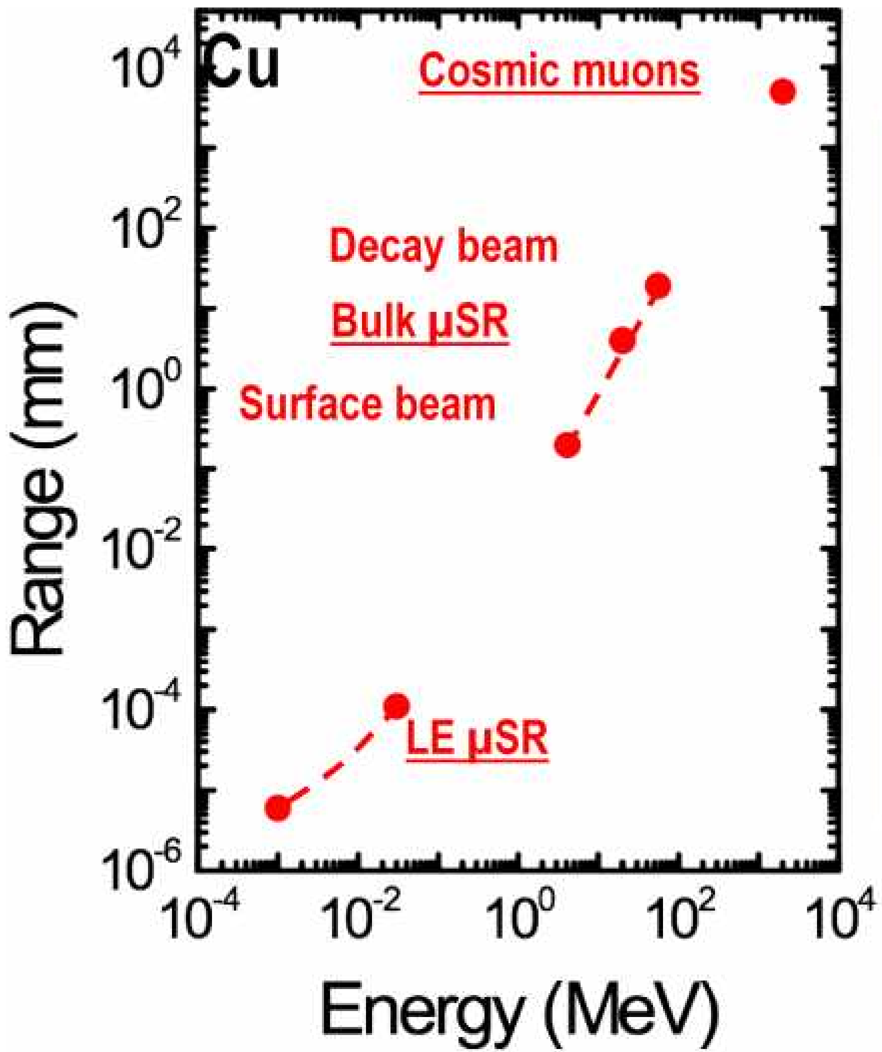
CeOFeAs structure:



Positive muon likes to stop:

- In the potential minimum
- High symmetry sites
- Near negative ions (e.g. O^{2-}, As^{3-}) (muon hydrogen bond like in OH with ~1 A bond length)
- Large spaces in the crystal structure

Muon Implantation Depth



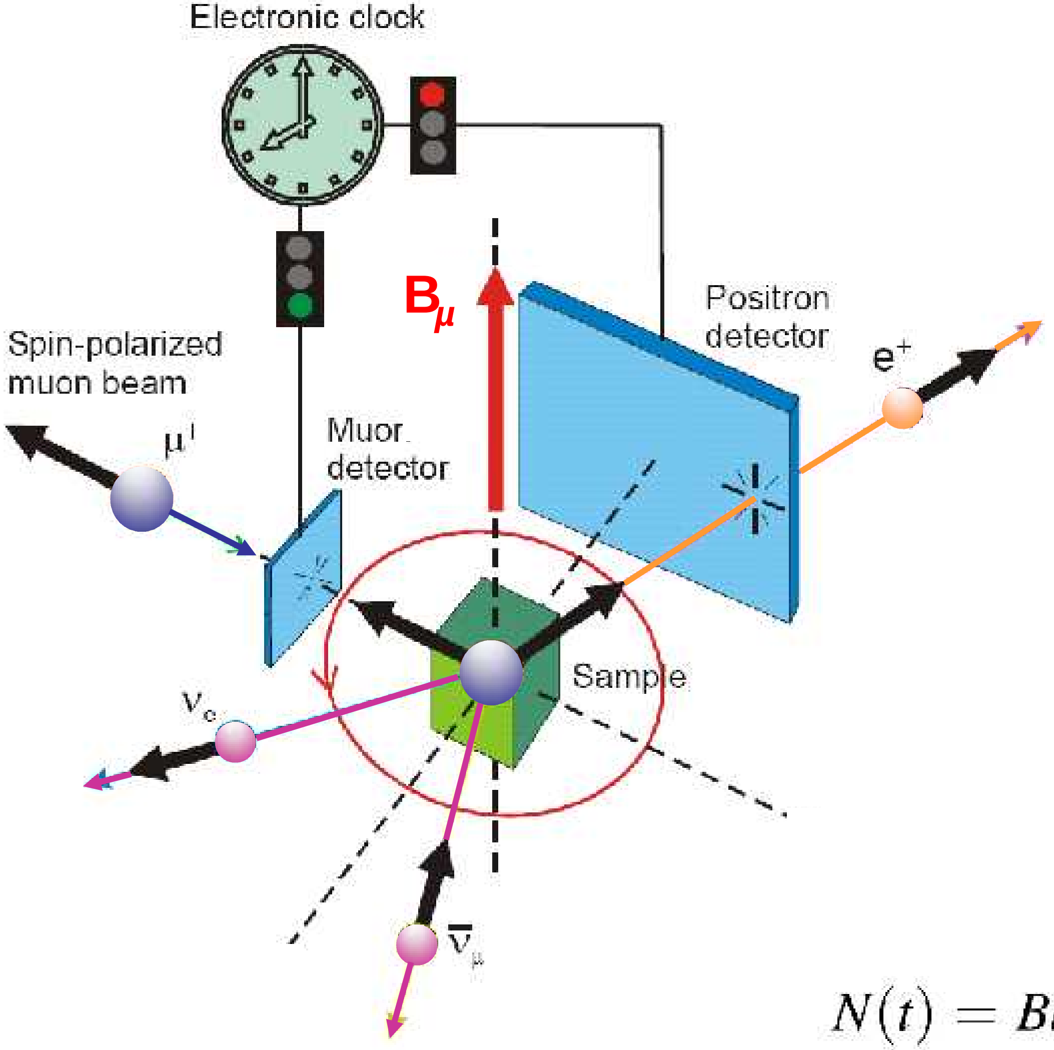
Bulk µSR:

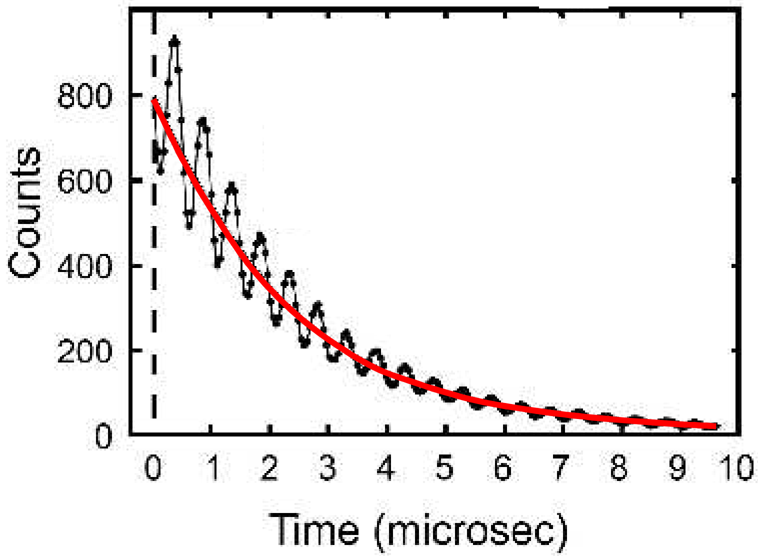
- *"Normal" samples (sub-mm)*
- *Bulky samples + samples in containers or pressure cells*

LE-µSR:

- *Depth-selective investigations (1–200 nm)*

μSR





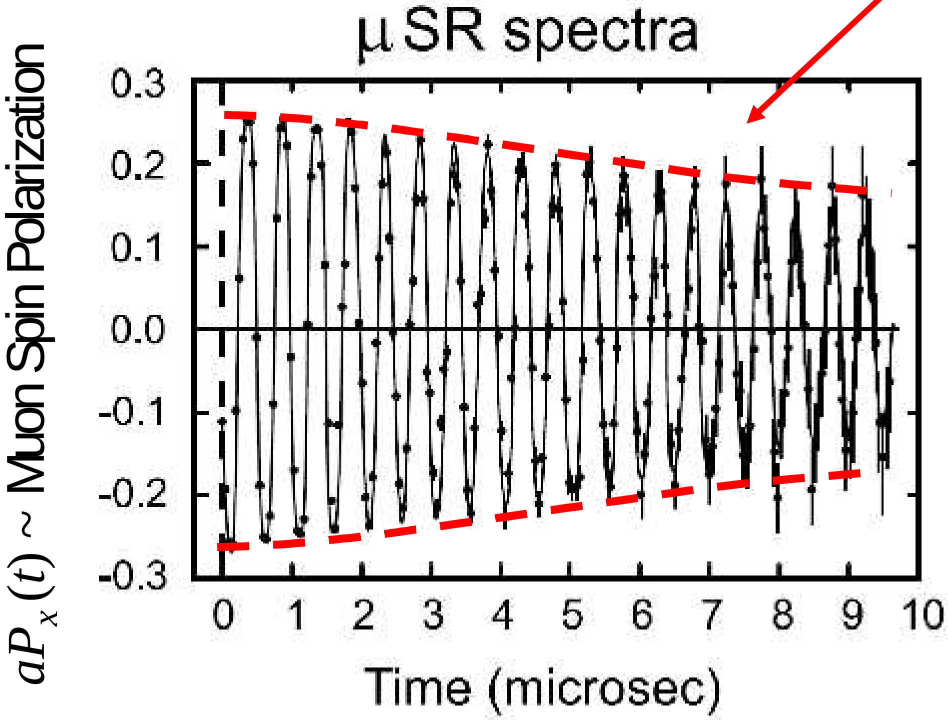
$$N(t) = Bkg + N_0 \exp(-t/\tau_\mu)$$

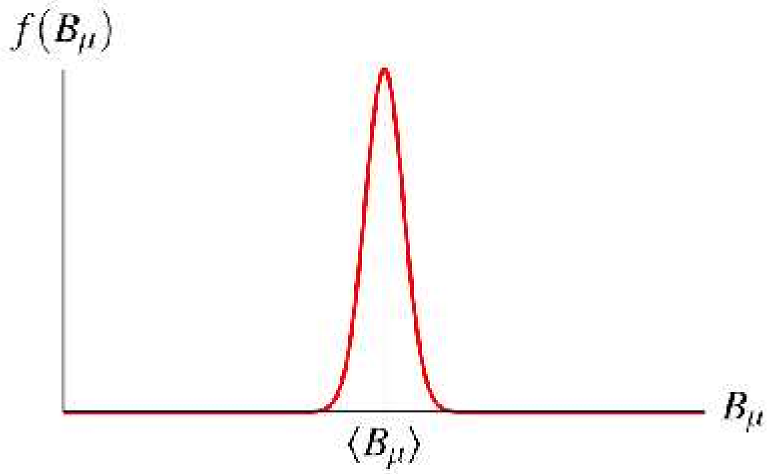
B_μ internal or external field

$\hat{\mathbf{n}}$: direction of detector

µSR Spectra

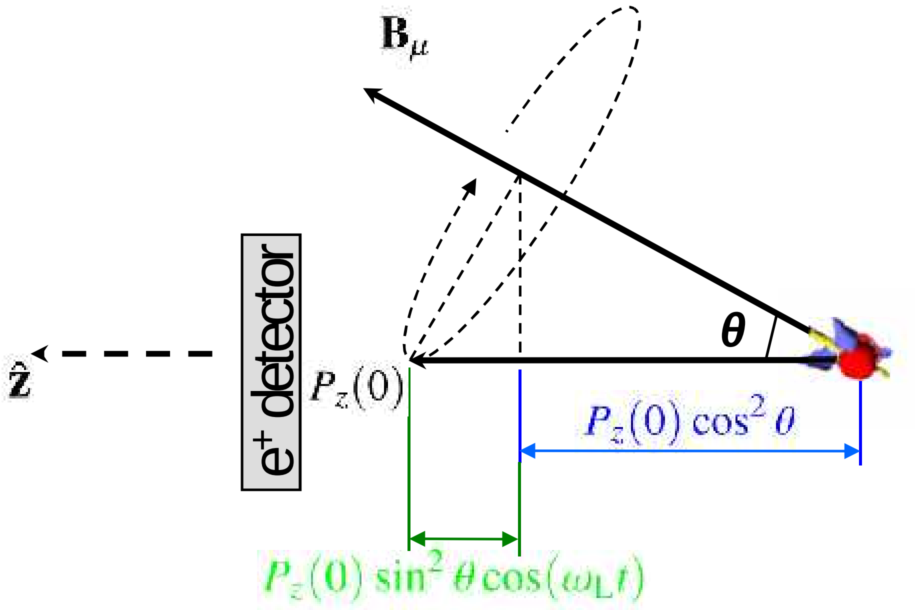
$$N(t) = Bkg + N_0 \exp(-t/\tau_\mu)\left[1 + \boxed{a\,\hat{\mathbf{n}}\cdot\mathbf{P}(t)}\right]$$





Frequency	▶	*Value of field at muon site* $(\omega_L = \gamma_\mu B_\mu)$
Damping	▶	*Field distribution and/or dynamics*

Field Direction -- Field Distribution

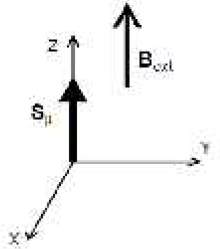


$$P_z(t) = \boxed{\cos^2\theta} + \boxed{\sin^2\theta\cos(\gamma_\mu B_\mu t)} \;)\;]\, d\mathbf{B}_\mu$$

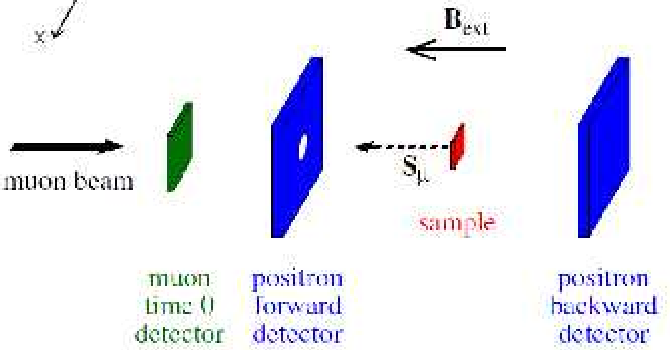
- Static part
- Oscillating part

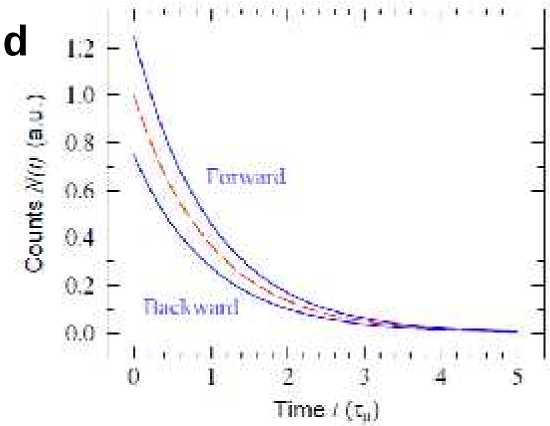
θ angle between magnetic field and muon polarization at $t = 0$

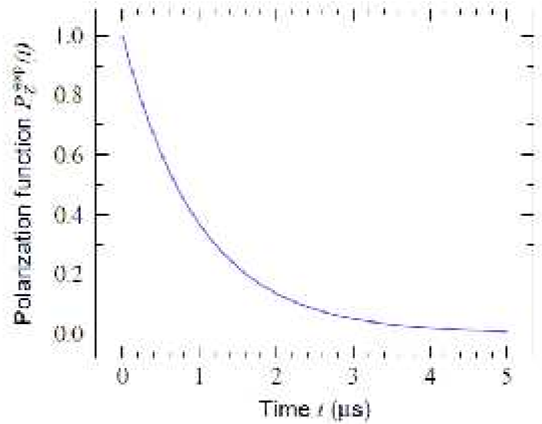
Different Measurement Geometries



ZF and LF: Zero field and Longitudinal Field geometry





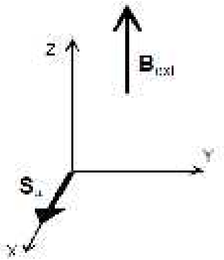


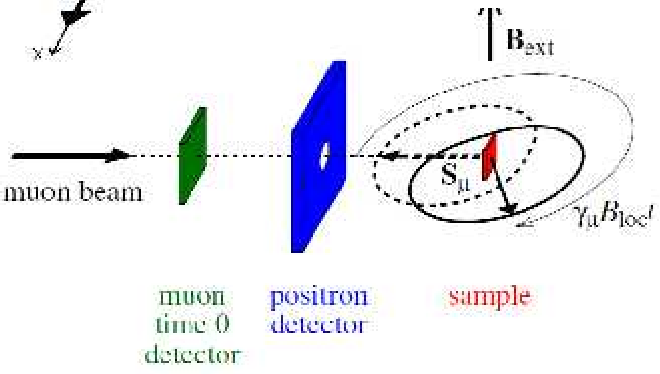
Typically used for the study of:

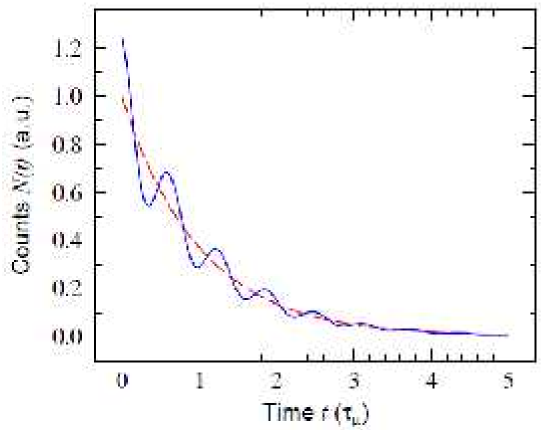
- Static magnetism
  - Temperature dependence of the magnetic order parameter
  - Determination of the magnetic transition temperature
  - Homogeneity of the sample
- Dynamic magnetism
  - Determination of magnetic fluctuation rates
  - Slowing down of fluctuations near phases transitions

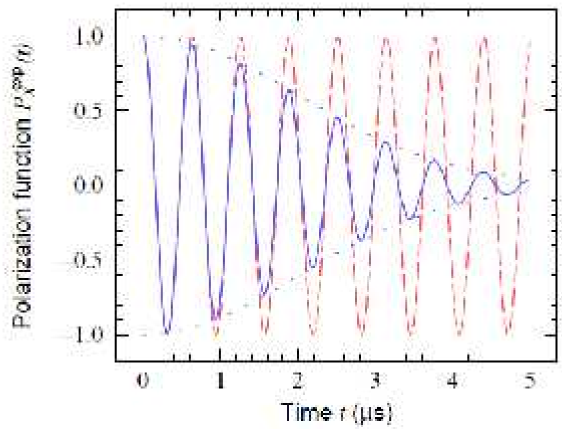
Different Measurement Geometries

TF: Transverse Field geometry









Typically used for the study of:

- Magnetism
  - Determination of the magnetic transition temperature
  - Homogeneity of the sample
  - Knight shift (local susceptibillity)
- Superconductivity
  - Absolute value and temperature dependence of the London penetration depth
  - Coherence length, votex structure, vortex dynamics, …

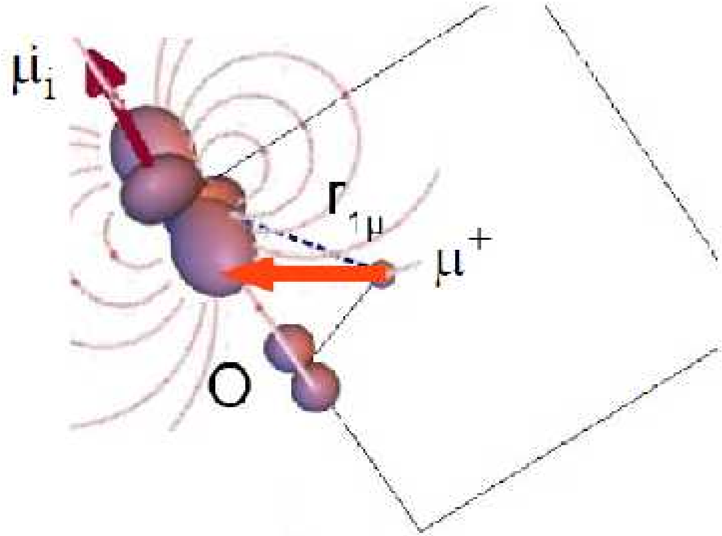
PAUL SCHERRER INSTITUT

Local field in magnetic materials

Internal field : generally sum of dipolar :

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{3(\vec{\mu}_i \cdot \vec{r}_{i\mu})\cdot r_{i\mu} - \vec{\mu}_i r_{i\mu}^2}{r_{i\mu}^5}$$

$$B_{dip} \approx \frac{\mu_0}{4\pi}\frac{\mu_i}{r_{i\mu}^3} \approx \frac{\mu_i[\mu_B]}{d^3[A^3]}\,T \approx 0.1T$$



and contact field (spin density at muon site):

$$\vec{B}_{hf}(\vec{r}_\mu) = \frac{2\mu_0}{3}\mu_B \rho_{spin}(\vec{r}_\mu) \simeq \frac{2\mu_0}{3}\mu_B \left|\varphi(\vec{r}_\mu)\right|^2 < \vec{s} >$$

High sensitivity:

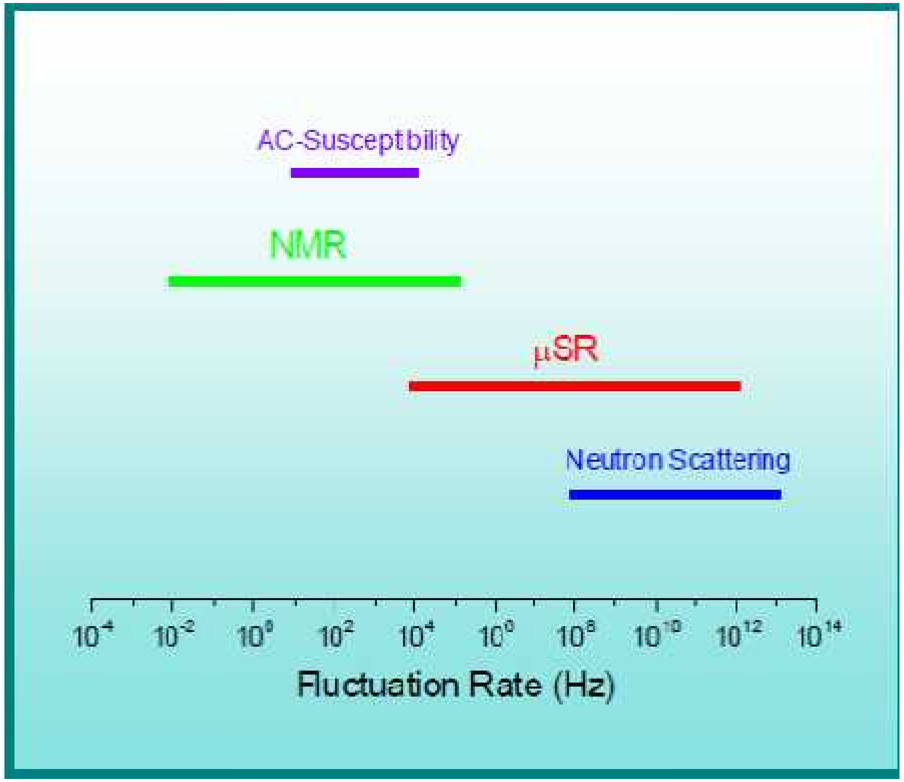
μSR time window 10-20 μs

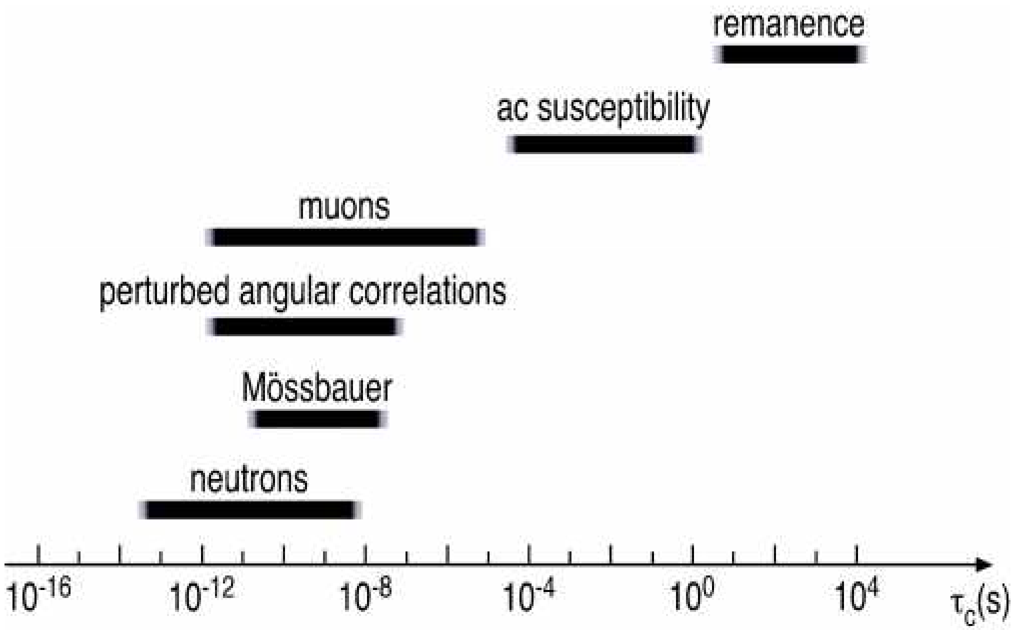
$\rightarrow \nu_\mu \simeq 50$ kHz detectable

$\rightarrow B = \frac{2\pi}{\gamma_\mu}\nu_\mu \approx 0.1$mT (Gauss)

(corresponds to $0.001\mu_B$ or nuclear moments μ_n)

The µSR technique has a unique time window for the study of magnetic flcutuations in materials that is complementary to other experimental techniques.



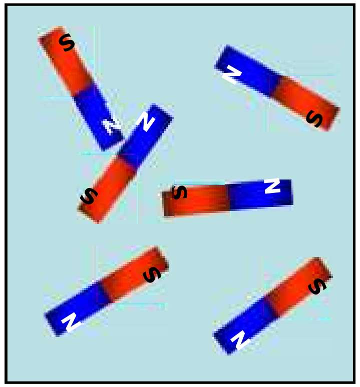


Muon Spin Rotation / Relaxation on Magnetic Materials

Magnetism

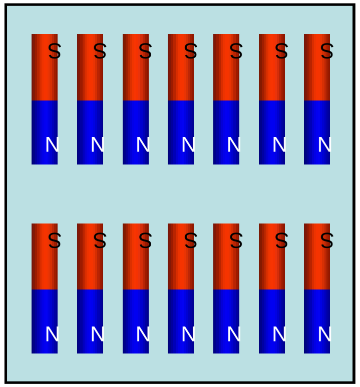
Magnetism

Paramagnetism



fluctuating
$T>T_C$

Ferromagnetism

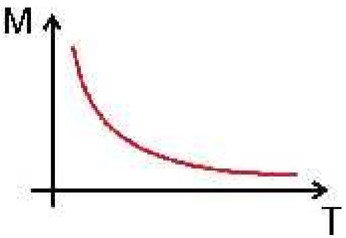


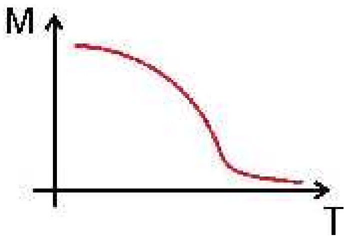
static
$T<T_C$

The interesting property of magnetically ordered system is the size and temperature dependence of the magnetic moment.

How do you measure this?

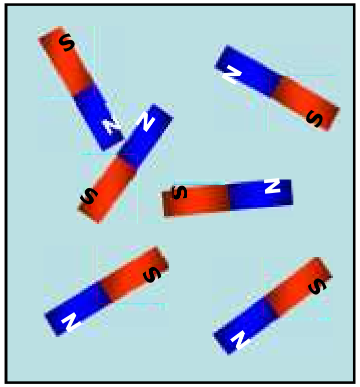
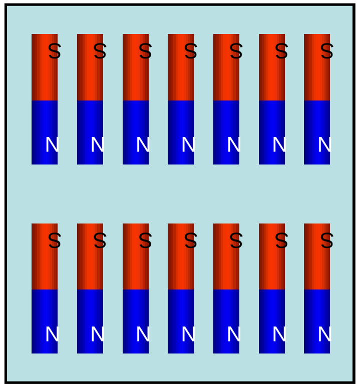
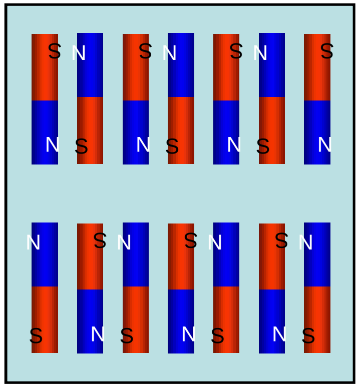
Macroscopic techniques (average over the hole sample):





SQUID, PPMS, …

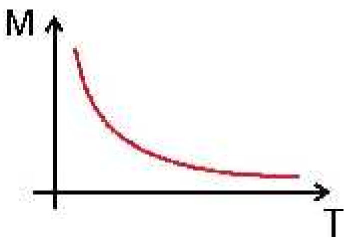
Magnetism

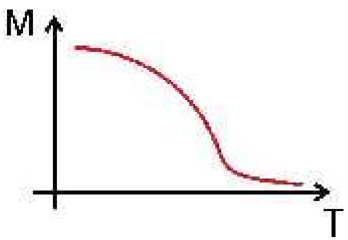
Paramagnetism	Ferromagnetism	Antiferromagnetism
		
fluctuating $T>T_C$	static $T<T_C$	static $T<T_N$

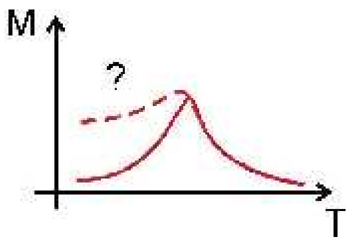
The interesting property of magnetically ordered system is the size and temperature dependence of the magnetic moment.

How do you measure this?

Macroscopic techniques (average over the hole sample):



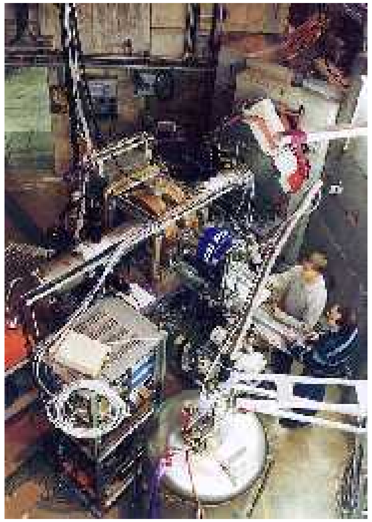




Magnetism

Scattering techniques:
(neutrons, X-rays)

Local probes:
(μSR, NMR, …)



Strength of muon spin rotation / relaxation:

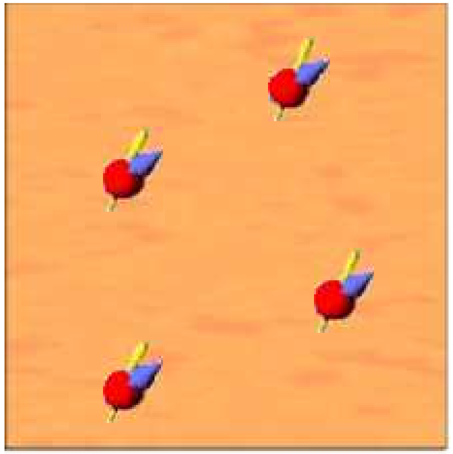
Investigation of magnetically inhomogeneous materials:

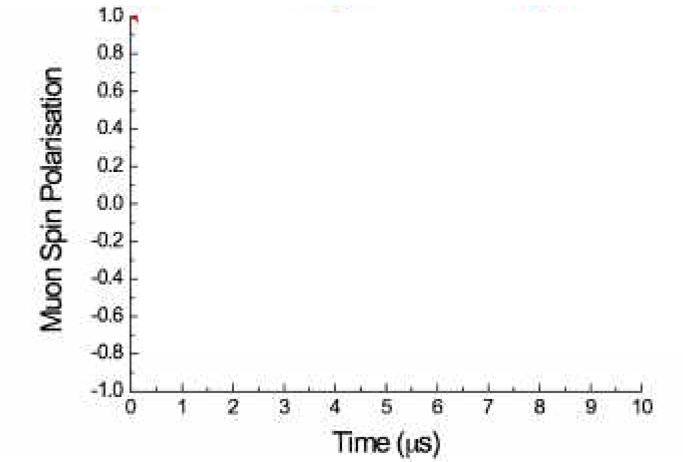
- Chemical inhomogeneity (“dirty samples”)
- **Competing interactions, coexistence of different magnetic orders, short range order, magnetic frustration**
- **Magnetism and superconductivity (competition and coexistence)**

Magnetically Inhomogeneous Materials

Homogen:

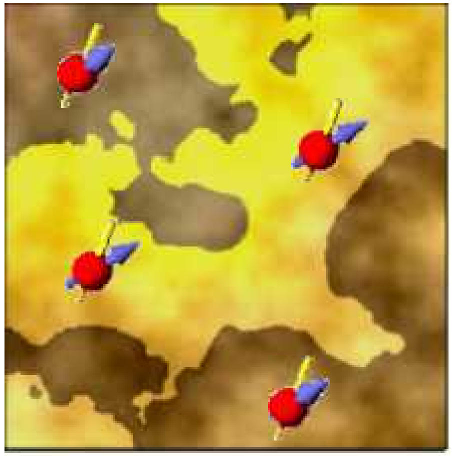
M_{hom}

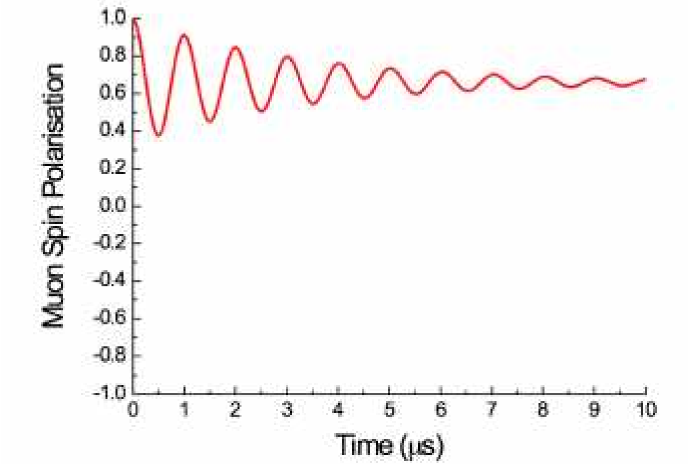




Inhomo-
geneous:

$M_{inhom} = M_{hom}$





Amplitude = Magnetic volume fraction
Frequency = Size of the magnetic moments (order parameter)
Damping = Inhomogeneity within the magnetic areas

Sensitivity of the Technique

Internal field at the muon site:

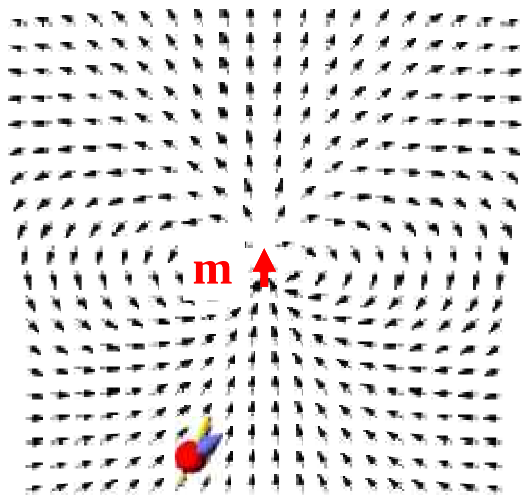
$$\mathbf{B}_\mu = \boxed{\mathbf{B}_c} + \boxed{\mathbf{B}_{\mathrm{dip}}}$$

- Contact field $\propto e|\Psi(\mathbf{r}_\mu)|^2$
- Dipolar contribution

$$\mathbf{B}_{\mathrm{dip}} = \sum_i \frac{1}{r_i^3}\left[\frac{(3\mathbf{m}_i \cdot \mathbf{r}_i)}{r_i^2}\mathbf{r}_i - \mathbf{m}_i\right]$$

$$B_{\mathrm{dip}} \simeq \frac{m}{r^3}$$

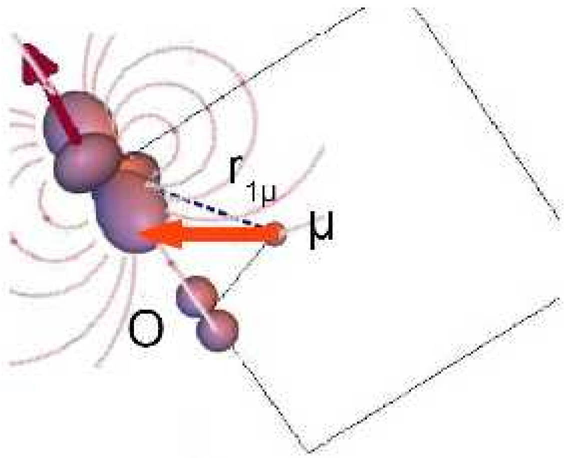
For $m = 1\,\mu_B$ and $r = 1\,\text{Å} \Rightarrow \; B_{\mathrm{dip}} \simeq 1\,\mathrm{T}$



➡ *static moments as low as 0.001 μ_B can be detected by µSR*

µSR time window: 10-20 µsec

➡ Frequencies down to 50 kHz detectable
Fields of few Gauss (10^{-4} T)



Advantages of µSR

Muons are **purely magnetic** probes (I = ½, no quadrupolar effects).

Local information, **interstitial** probe complementary to NMR.

Large magnetic moment: $\mu_\mu = 3.18\ \mu_p = 8.89\ \mu_n$ **sensitive** probe.

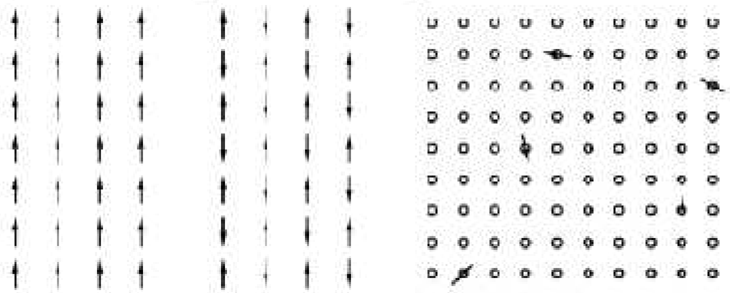
Particularly suitable for:
Very weak effects, small moment magnetism ~ $10^{-3}\ \mu_B$/Atom
Random magnetism (e.g. spin glasses).
Short range order (where neutron scattering is not sensitive).
Independent determination of **magnetic moment** and of **magnetic volume fraction.**

Determination of magnetic/non magnetic/superconducting fractions.

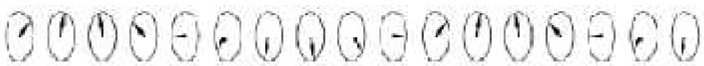
Full polarization in zero field, independent of temperature unique measurements without disturbance of the system.

Different Depolarization Functions

ferro antiferro dilute



helical



spiral

- nuclear moments
- electronic moments

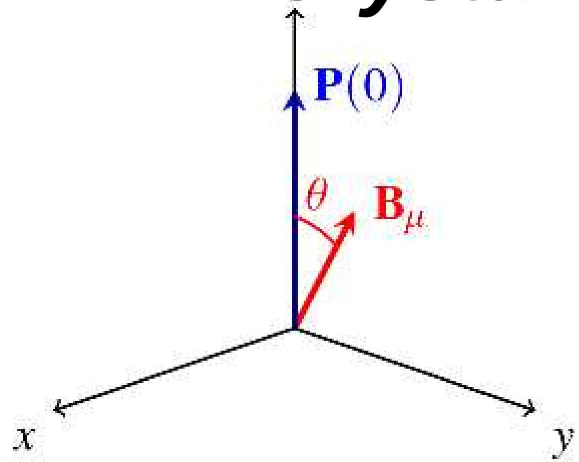
- static moments
- fluctuating moments

- various spin structures
- spin glasses (randomness)
- variety of field distributions

Magnetism of Single Crystals

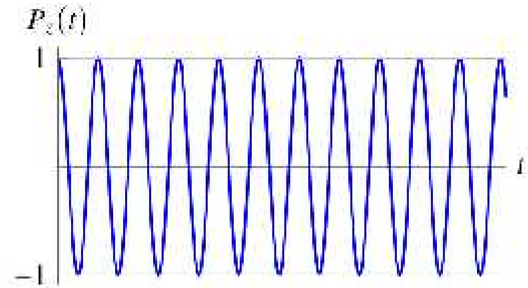
Simple Magnetic Sample – Single Crystal

$$P_z(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right] d\mathbf{B}_\mu$$



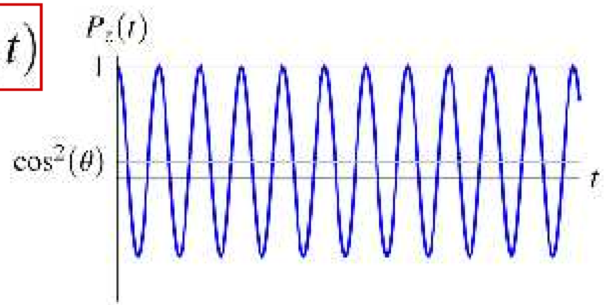
Single Crystal with $\theta = \pi / 2$

$$\boxed{P_z(t) = \cos(\gamma_\mu B_\mu t)}$$



Single Crystal with $\theta \neq \pi / 2$

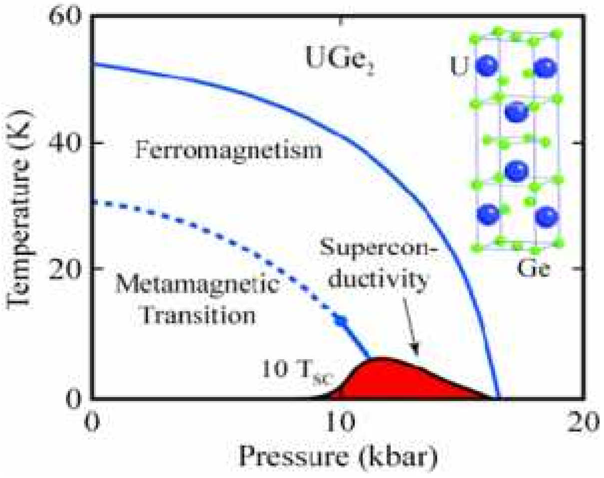
$$\boxed{P_z(t) = \cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)}$$



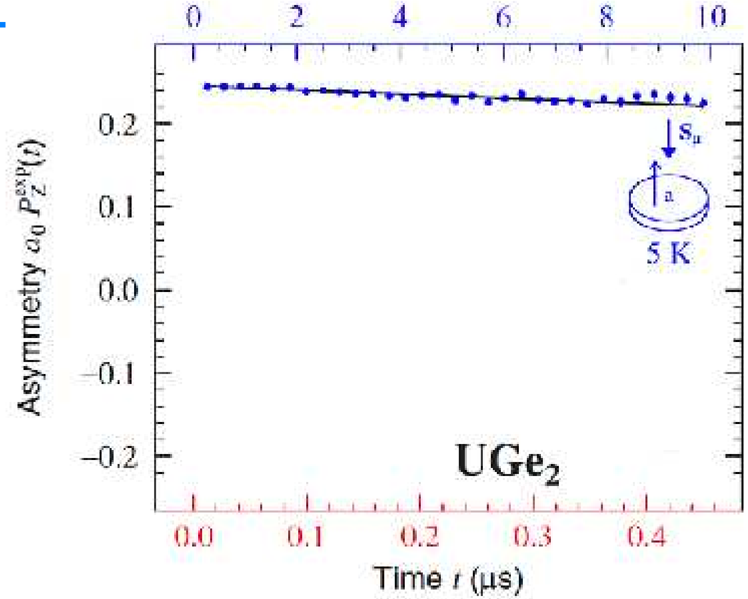
Þ in a single crystal the amplitude of the oscillatory component indicates the direction of the internal field

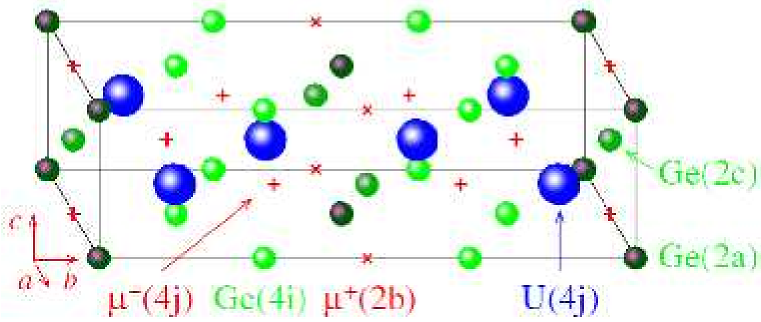
Example:
Single Crystalline UGe_2

Single Crystalline UGe_2



Cavendish Lab.





- Two magnetically inequivalent muon stopping sites
- No oscillations for $\mathbf{P}(0)$ pointing along *a*-axis
  Þ Internal fields parallel to *a*-axis ($\theta = 0$)
- Oscillations around zero
  Þ because single crystal

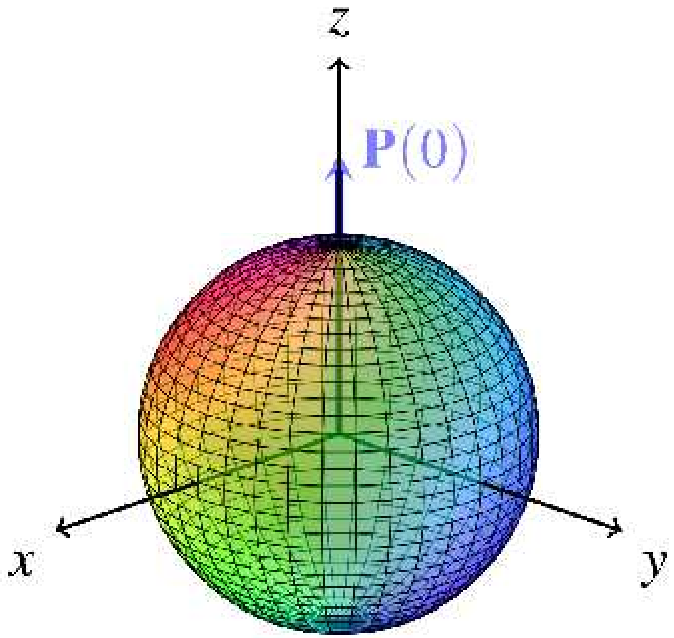
S. Sakarya et al., Phys. Rev. B 81, 024429 (2010)

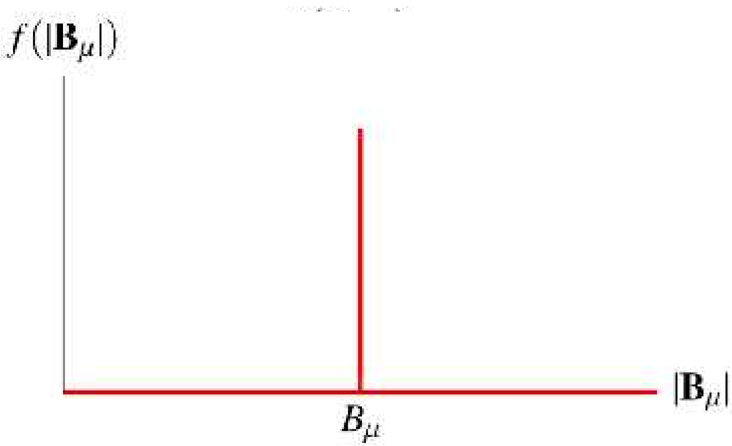
Magnetism of Polycrystals (Powders)

Simple Magnetic Sample – Polycrystal

$$P_z(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right]d\mathbf{B}_\mu$$

Polycrystal (powder)





If isotropic:

$$f(|\mathbf{B}_\mu|) = f(\mathbf{B}_\mu)\,4\pi B_\mu^2$$

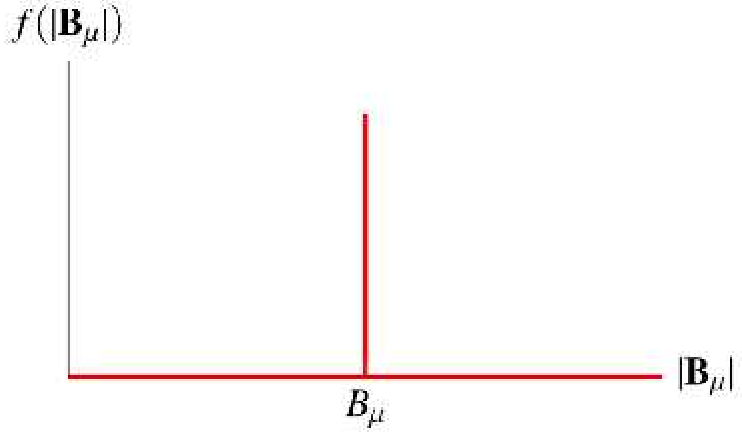
$$f(\mathbf{B}_\mu)\,d\mathbf{B}_\mu = \frac{f(|\mathbf{B}_\mu|)}{4\pi}\sin(\theta)\,d\theta\,d\phi\,dB_\mu$$

$$\boxed{P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma_\mu B_\mu t)}$$

Simple Magnetic Sample – Polycrystal

$$P_z(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right]d\mathbf{B}_\mu$$

Polycrystal: Ideal Case

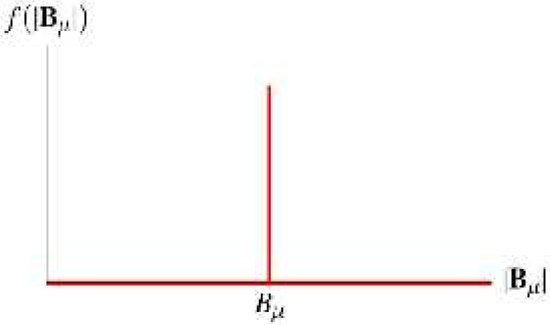


$$\boxed{P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma_\mu B_\mu t)}$$

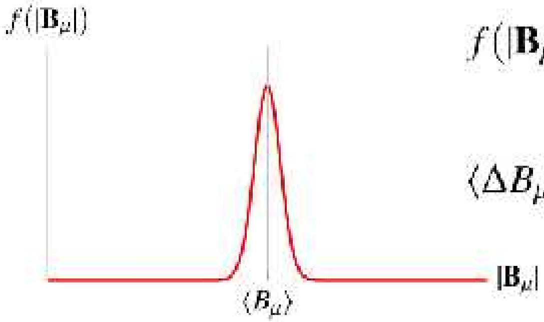
Simple Magnetic Sample – Polycrystal

$$P_z(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right]d\mathbf{B}_\mu$$

Polycrystal: Ideal Case ► **Polycrystal: Real Case**



►



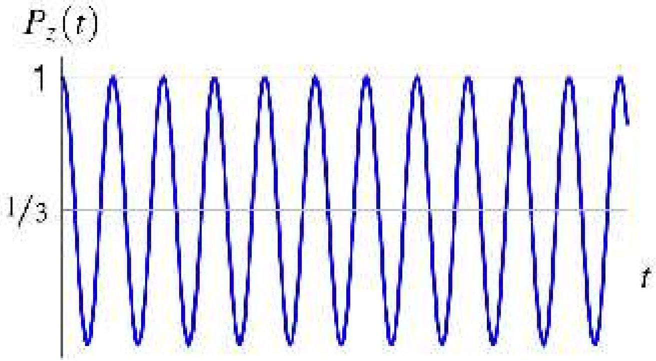
$$f(|\mathbf{B}_\mu|) = \frac{1}{\sqrt{2\pi\langle\Delta B_\mu^2\rangle}}\exp\left[-\frac{(B_\mu - \langle B_\mu\rangle)^2}{2\langle\Delta B_\mu{}^2\rangle}\right]$$

$$\langle\Delta B_\mu{}^2\rangle = \int (B_\mu - \langle B_\mu\rangle)^2 f(|\mathbf{B}_\mu|)\,dB_\mu$$

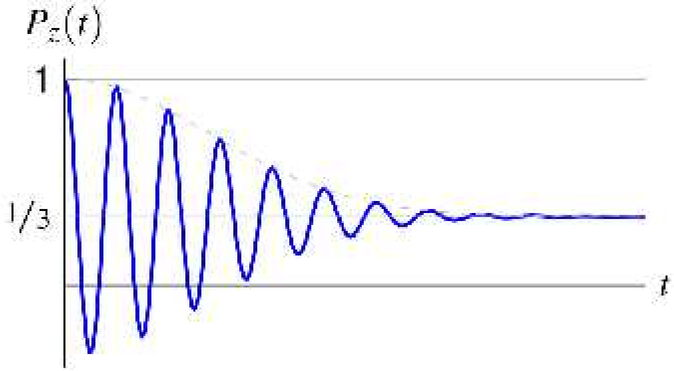
$$\boxed{P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma_\mu B_\mu t)}$$

►

$$\boxed{P_z(t) = \frac{1}{3} + \frac{2}{3}\exp\left[-\frac{1}{2}\gamma_\mu^2\langle\Delta B_\mu{}^2\rangle t^2\right]\cos\left[\gamma_\mu\langle B_\mu\rangle t\right]}$$



►



Example:

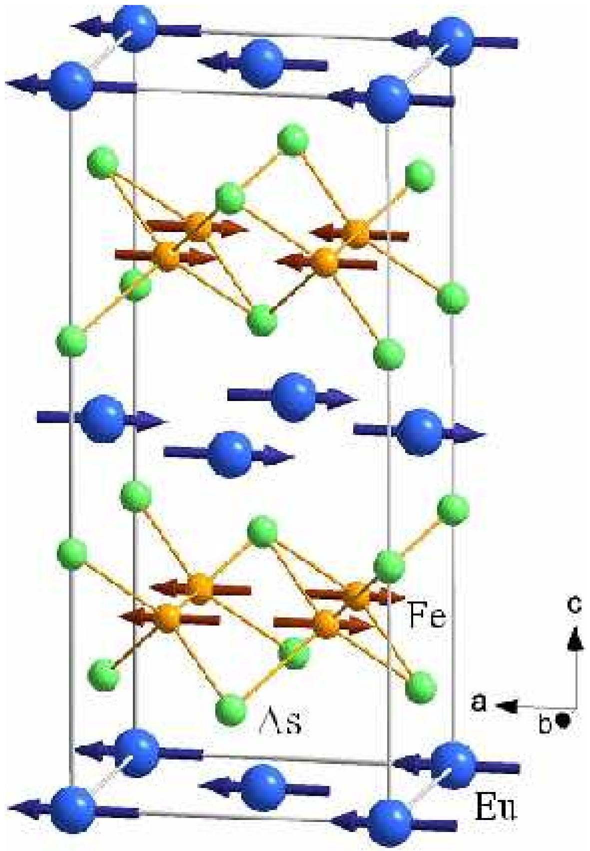
Magnetism of Polycrystalline $EuFe_2(As_{1-x}P_x)_2$

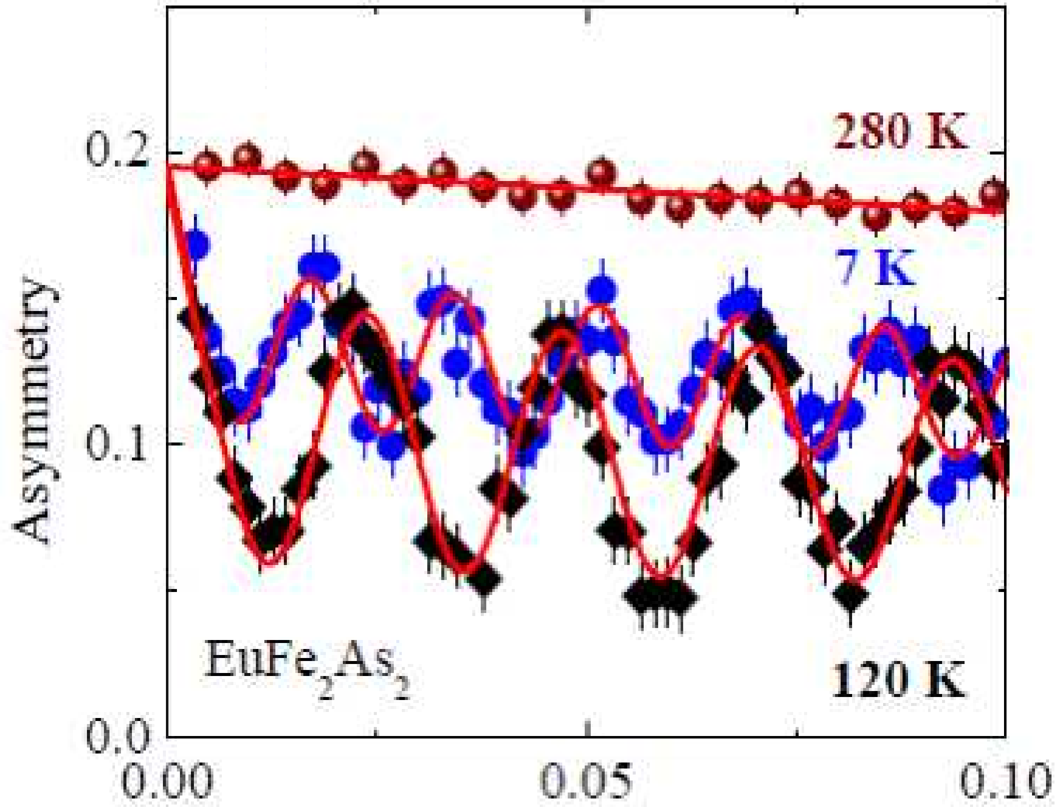
$A Fe_2As_2$(A=Ba,Sr,Ca,Eu)

T_{SDW}(Fe) = 190 K

Eu^{2+} $4f^{7}$, S = 7/2

$T_{AFM}(Eu^{2+})$ = 19 K



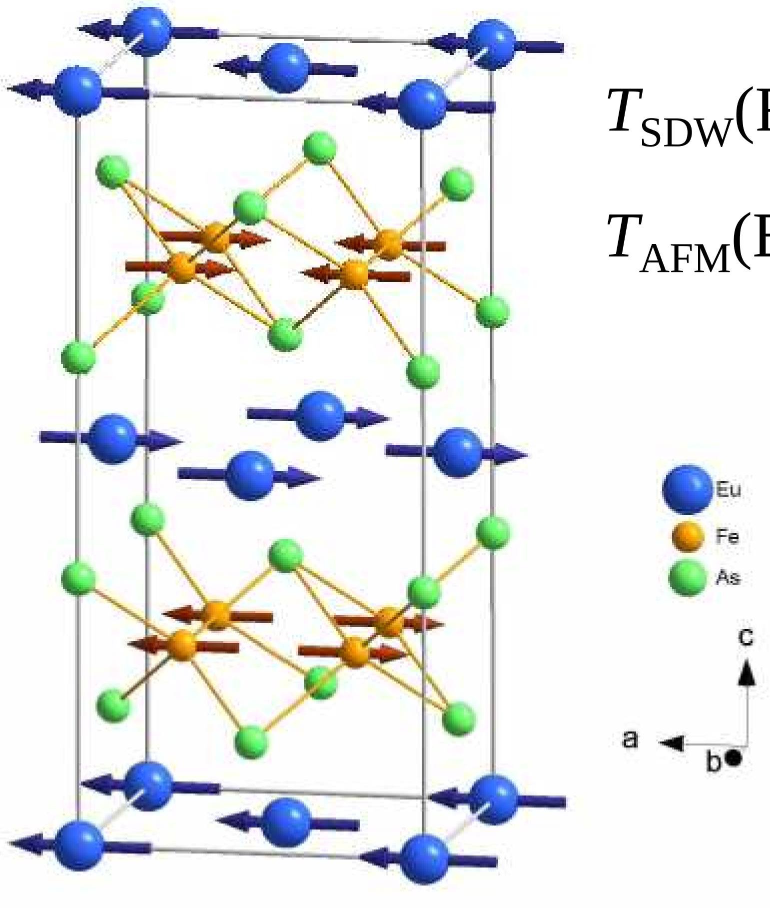


Guguchia *et.al,* PRB **83**, 144516 (2011).

Magnetic structure of $EuFe_2As_2$ at 2.5 K

AFe$_2$As$_2$(A=Ba,Sr,Ca,Eu)

$Eu^{2+} \rightarrow 4f^{7}$, S = 7/2

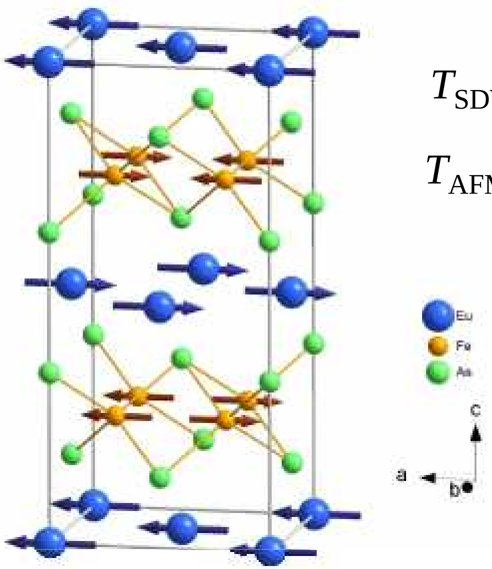


T_{SDW}(Fe) = 190 K

$T_{AFM}(Eu^{2+})$ = 19 K

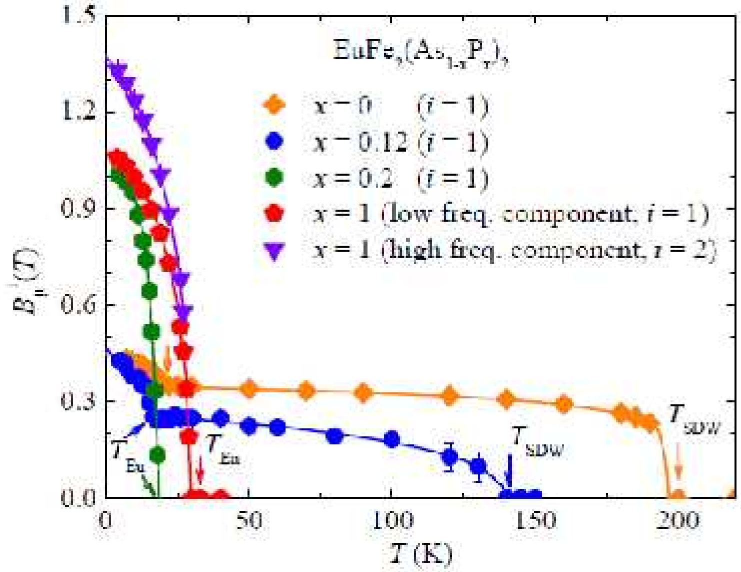
Y.Xiao *et al*., PRB 80, 174424 (2009).

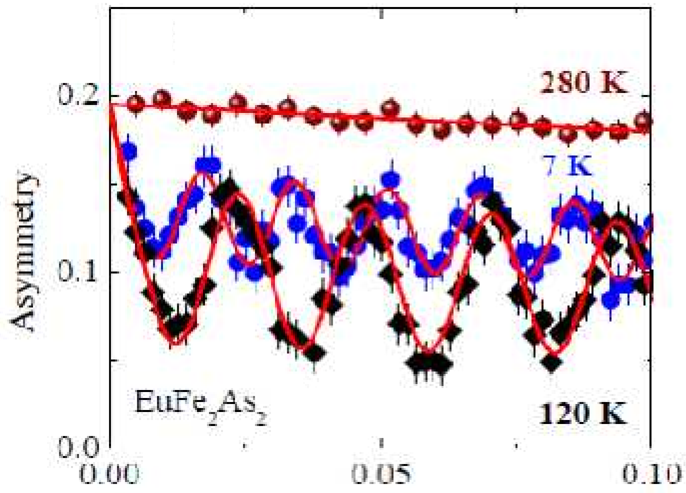
Raffius *et al*., J. Phys. Chem. Solids 54, 13 (1993).

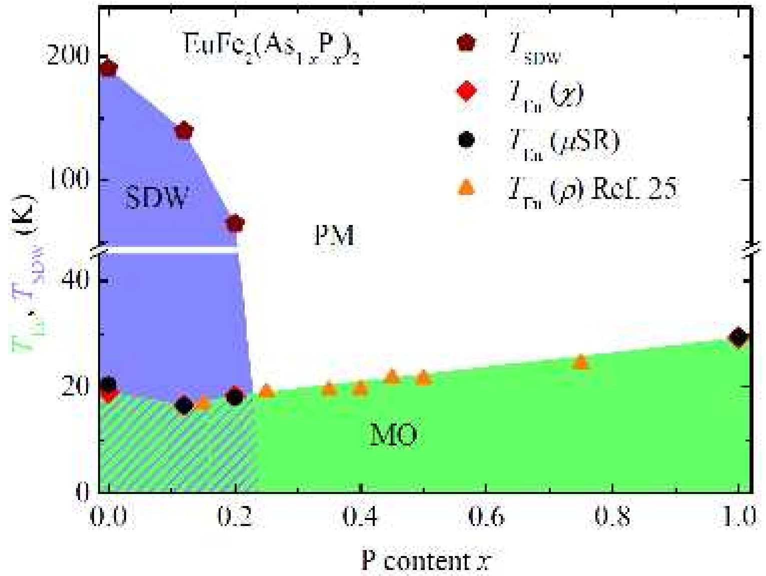


T_{SDW}(Fe) = 190 K

T_{AFM}(Eu^{2+}) = 19 K





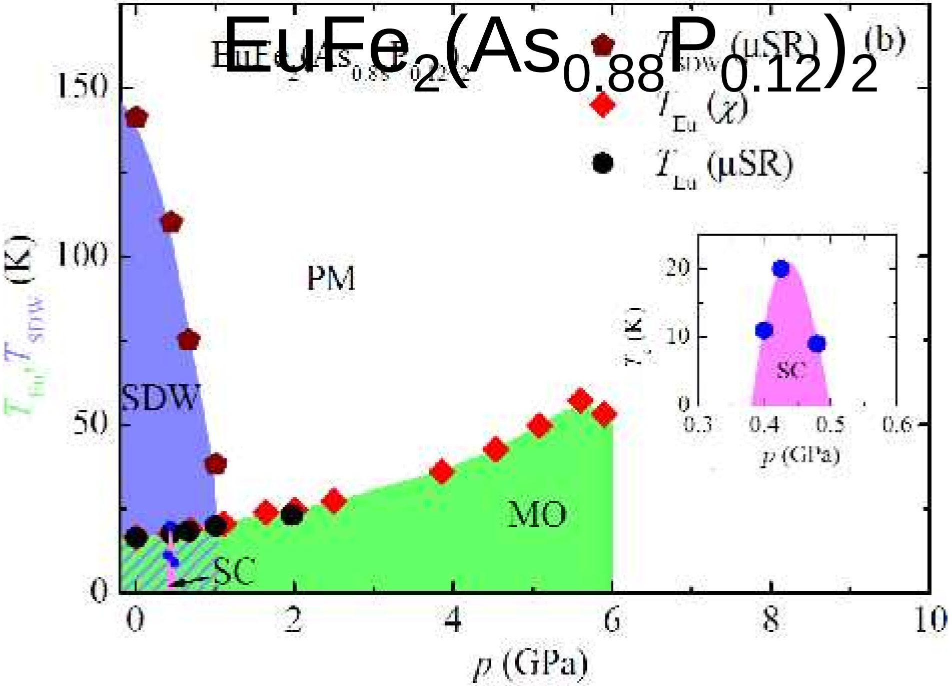


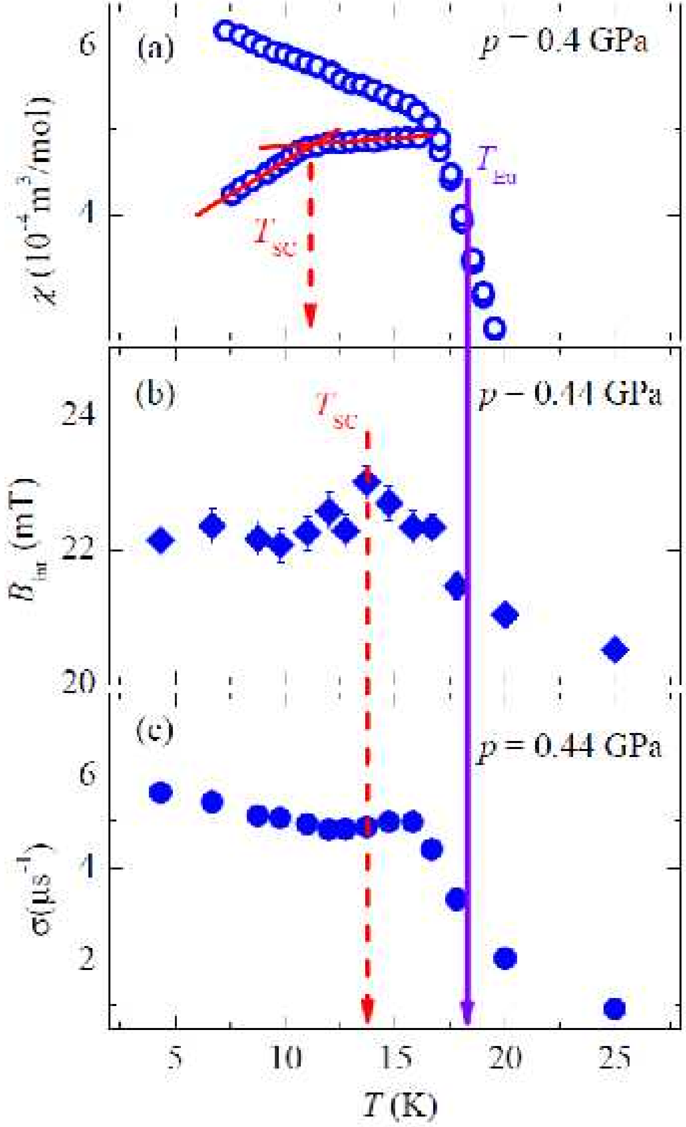
Z. Guguchia *et. al.,* Phys. Rev. B **83**, 144516 (2011).

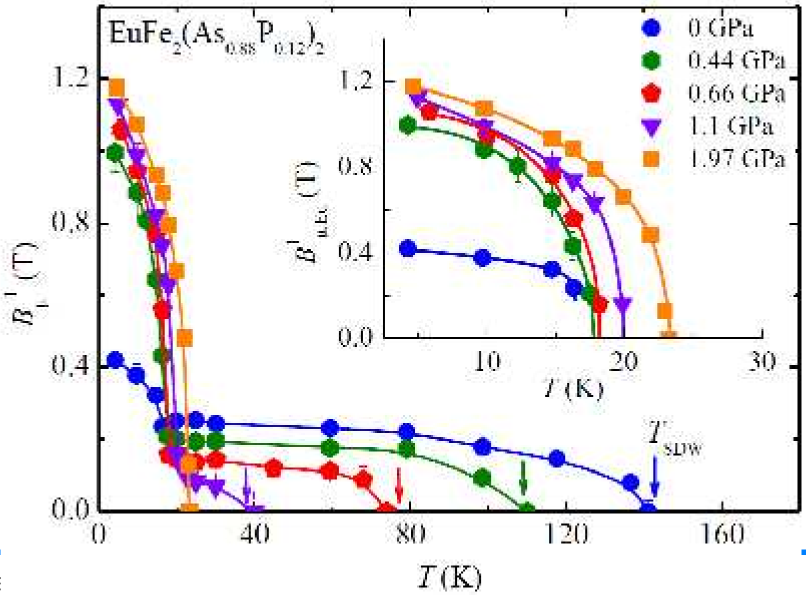
Y. Xiao *et al*., PRB 80, 174424 (2009).

Z. Guguchia, A. Shengelaya *et. al*., arXiv:1205.0212v1.

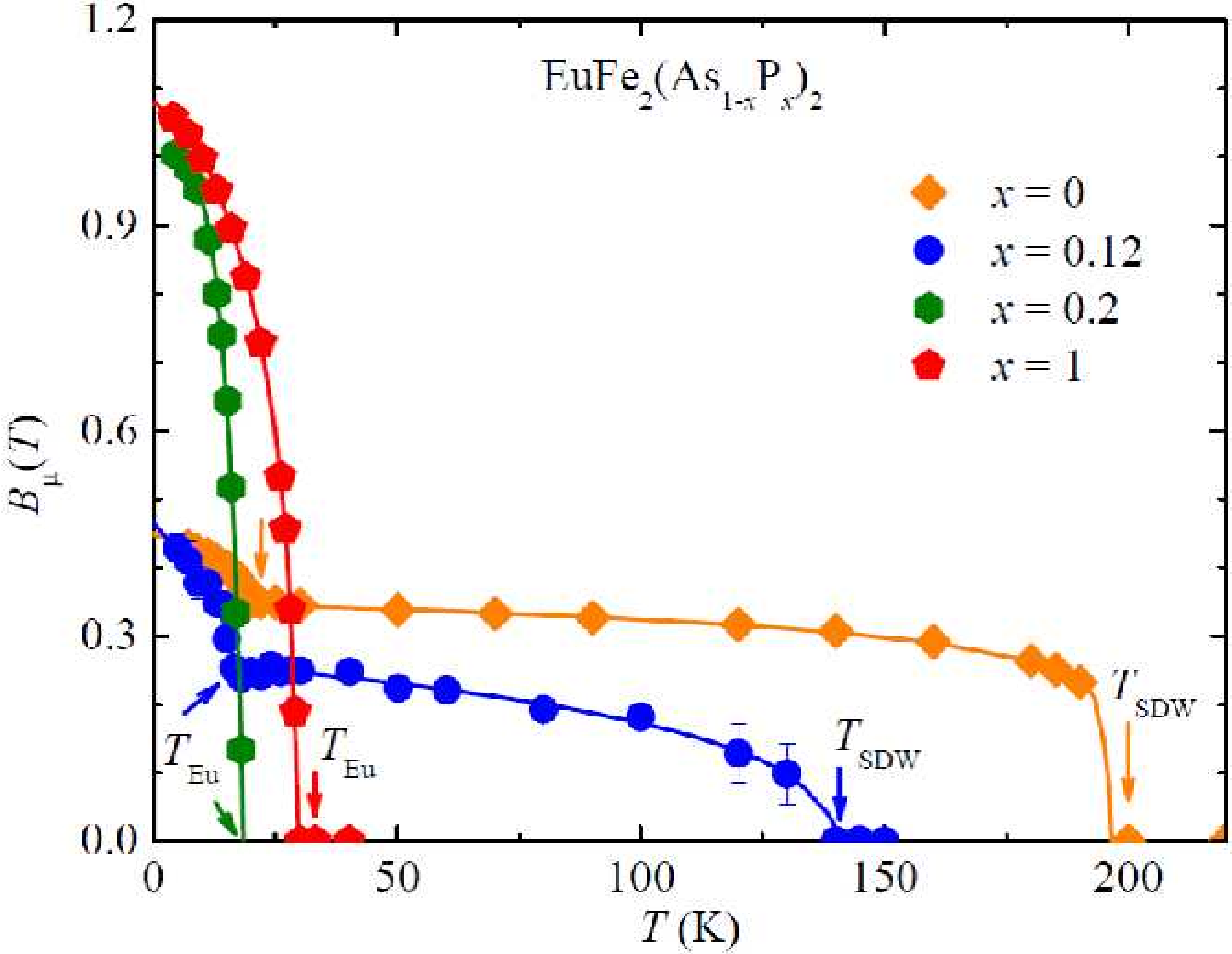
Temperature-pressure phase diagram for $EuFe_2(As_{0.88}P_{0.12})_2$





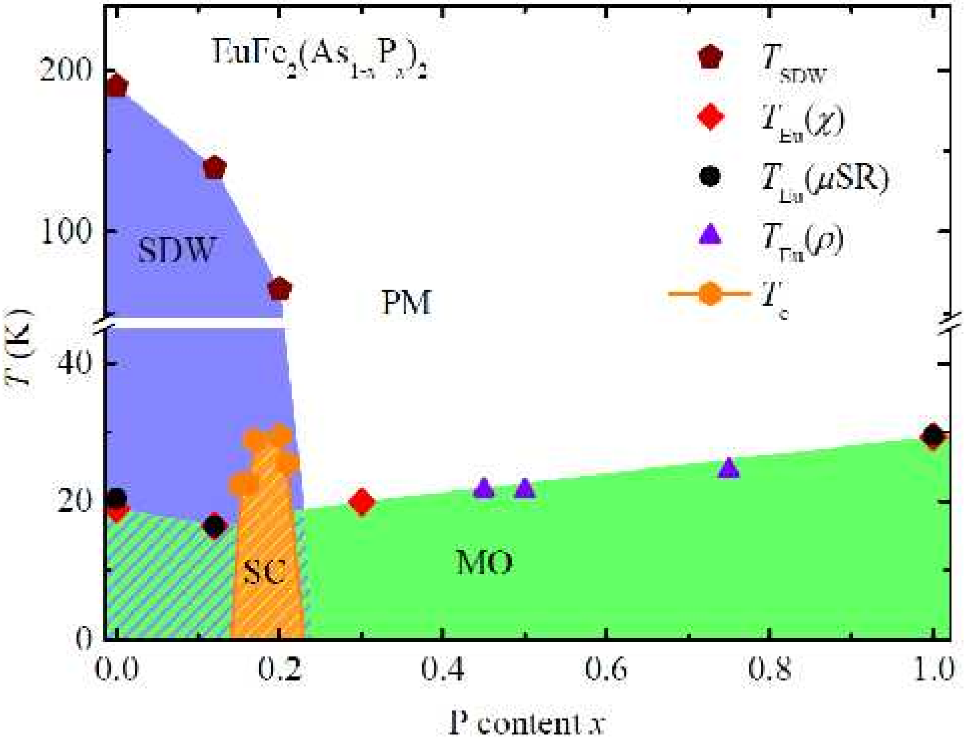


Z. Guguchia *et. al.*, arXiv:1205.0212v1.

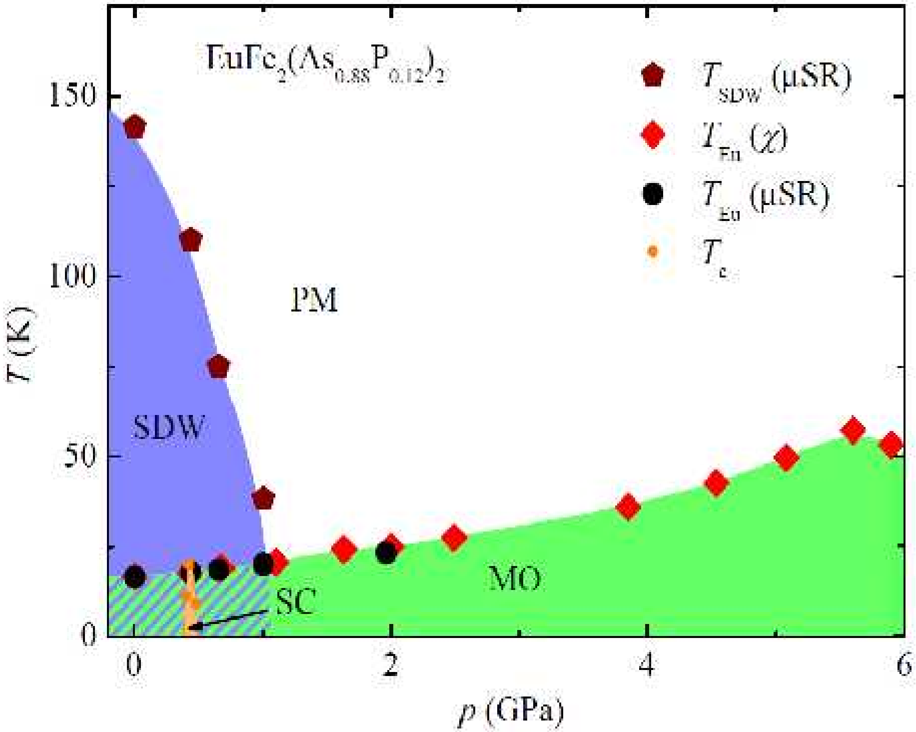


Guguchia *et al,* JSNM **26**, 285 (2013).

Temperature-doping phase diagram



Temperature-pressure phase diagram

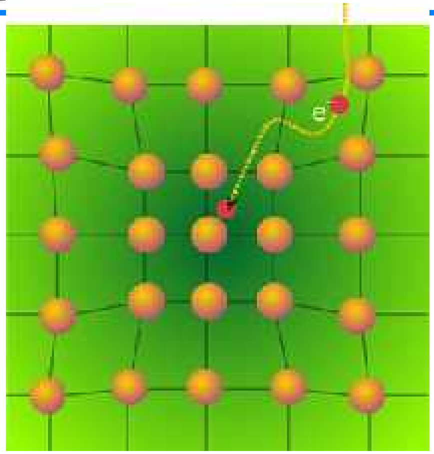


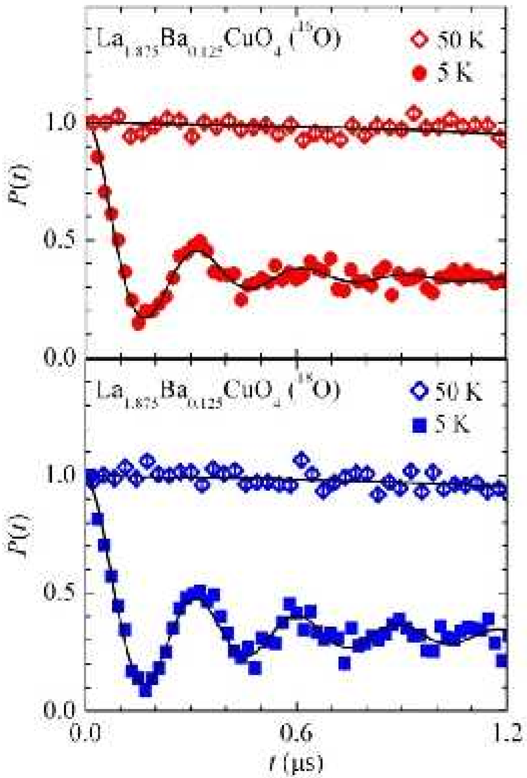
Guguchia *et al,* JSNM **26**, 285 (2013).

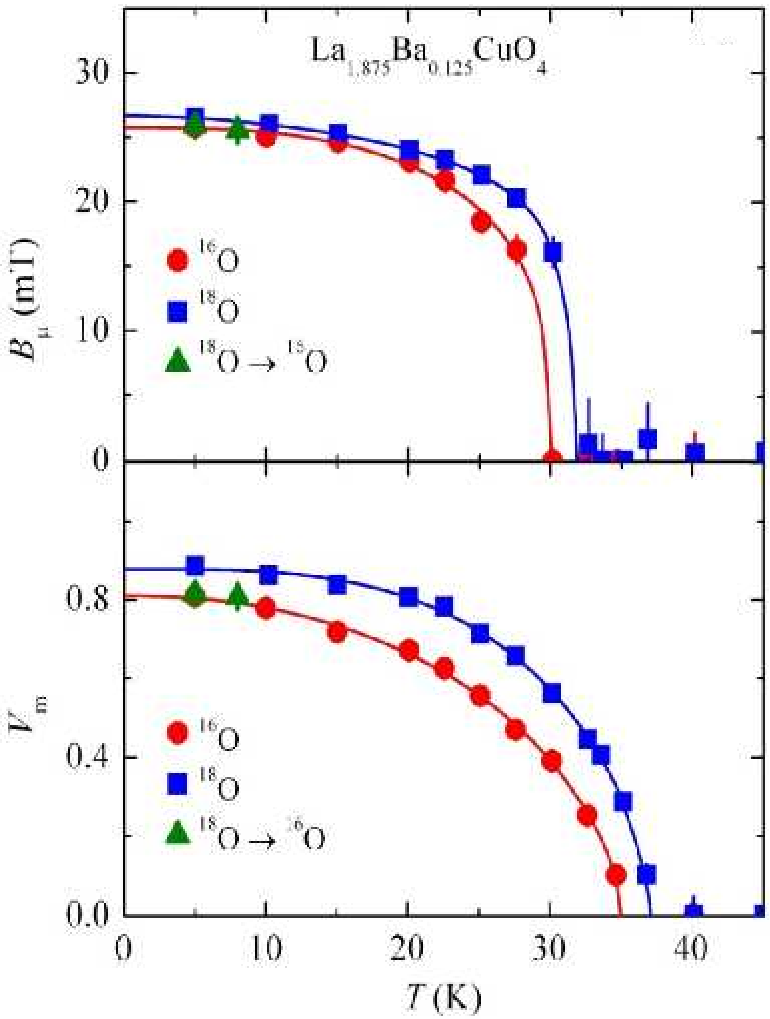
Example:

Magnetism of Polycrystalline $La_{1.875}Ba_{0.125}CuO_4$

OIE effect on T_{so} and magnetic fraction V_m



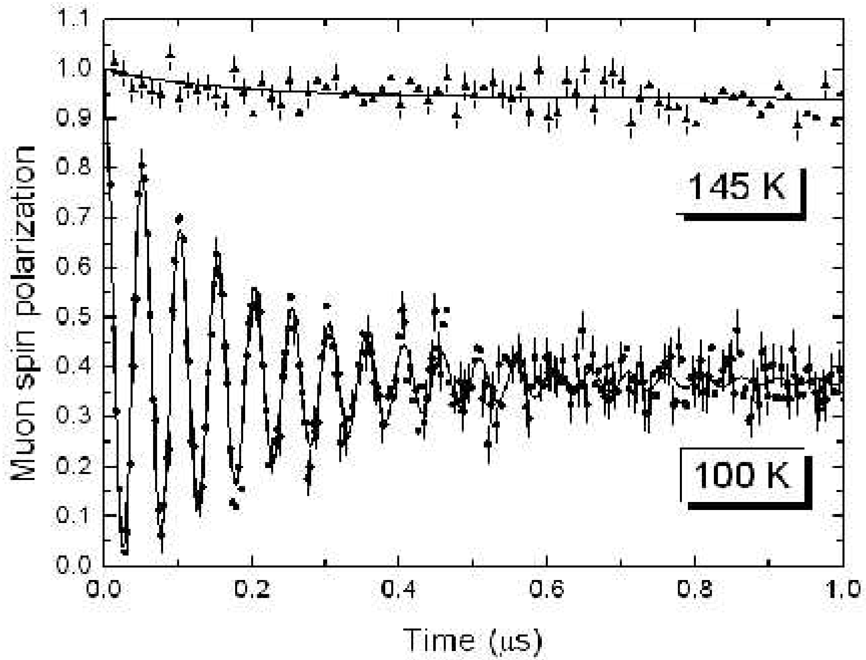




Z. Guguchia *et al.*, Phys. Rev. Lett. 113, 057002 (2014).

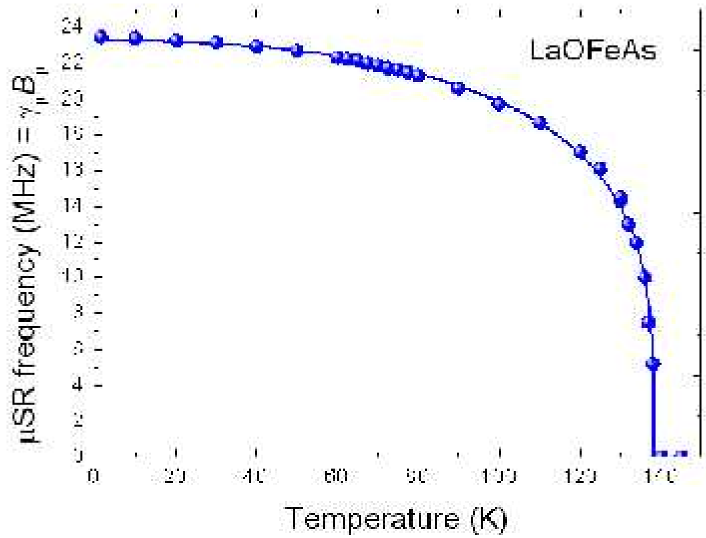
Example:
Magnetism of Polycrystalline LaOFeAs

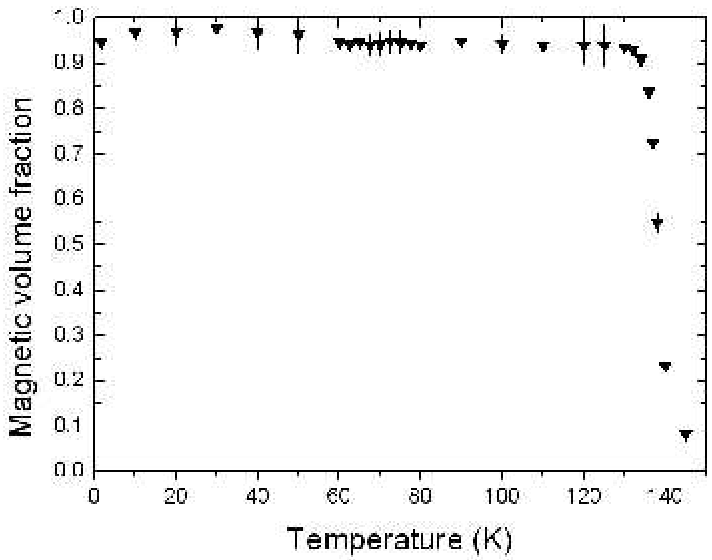
Polycrystalline LaOFeAs



- **Zero Field Muon Spin Rotation**
  - Static commensurate magnetic order

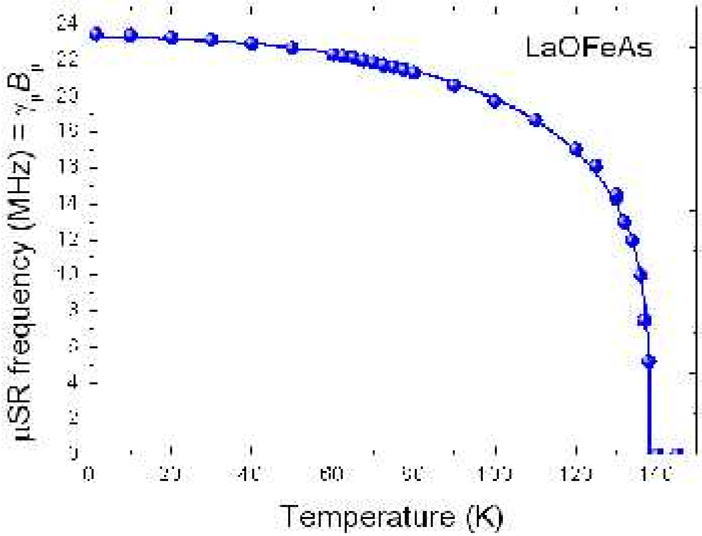
Polycrystalline LaOFeAs

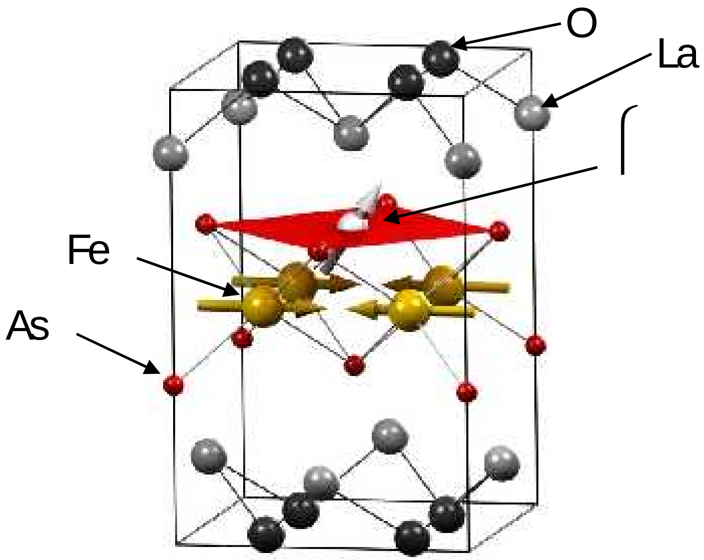




- **Zero Field Muon Spin Rotation**
  - Static commensurate magnetic order
  - T-dependence of the Fe magnetization with high precision, $T_N = 138$ K
  - 100% magnetic volume fraction

Polycrystalline LaOFeAs





- **Zero Field Muon Spin Rotation**
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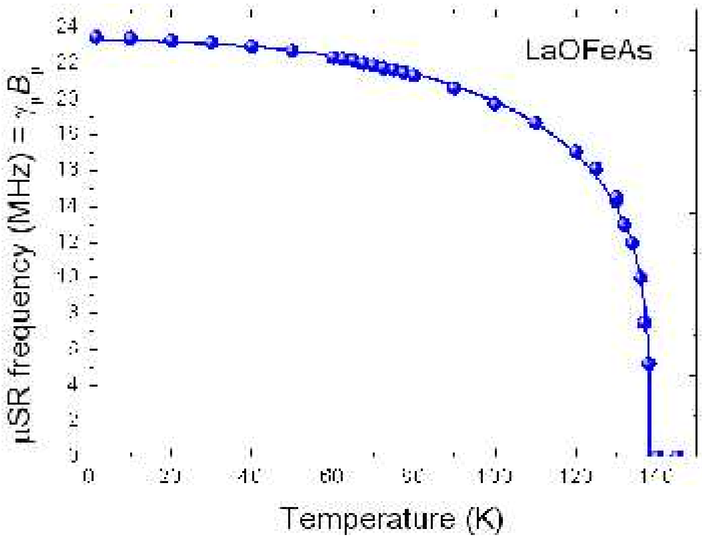
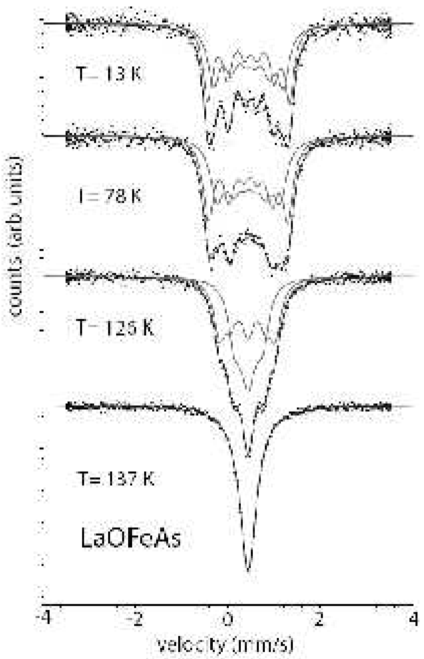
Polycrystalline LaOFeAs





- **Zero Field Muon Spin Rotation**
  - Static commensurate magnetic order
  - T-dependence of the Fe magnetization with high precision, $T_N = 138$ K
  - 100% magnetic volume fraction

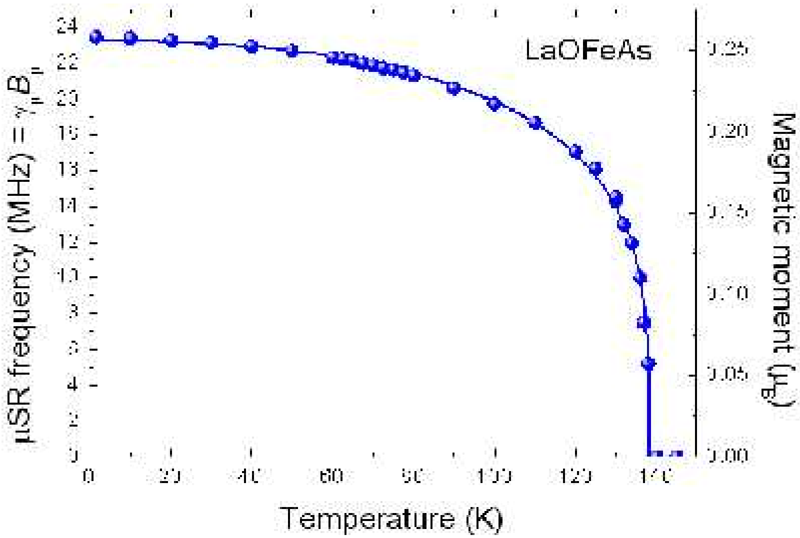
$$B_\mu = B_{\mathrm{dip}} + B_c + \dots$$

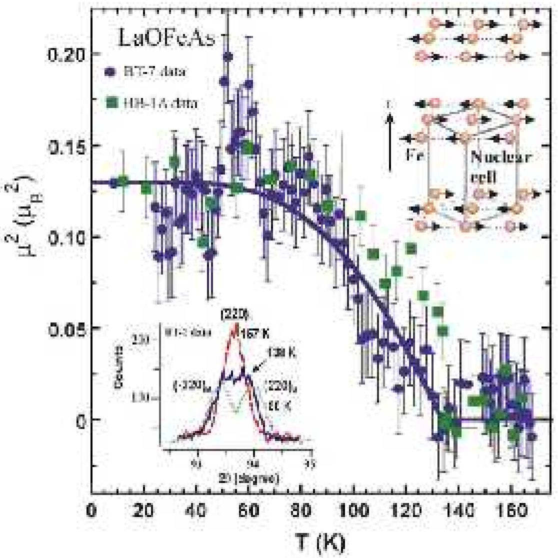
contact field $\propto c|\Psi(x_\mu)|^2$

$$B_{\mathrm{dip}} = \sum_i \frac{1}{r_i^3}\left[\frac{(3m_i \cdot r_i)}{r_i^2} r_i - m_i\right]$$

- **Mössbauer spectroscopy**
  - Hyperfine field $B_{hf}(0) = 4.86(5)$ T
    =>
    Saturation moment **$\mu = 0.25 - 0.32\ \mu_B$**

Polycrystalline LaOFeAs





C. de la Cruz *et al.*, Nature 453, 899 (2008)

- **Zero Field Muon Spin Rotation**
  - Static commensurate magnetic order
  - T-dependence of the Fe magnetization with high precision, T_N = 138 K
  - 100% magnetic volume fraction

$$B_\mu = B_{\mathrm{dip}} + B_c + \dots$$

contact field

$$\propto c|\Psi(x_\mu)|^2$$

$$B_{\mathrm{dip}} = \sum_i \frac{1}{r_i^3}\left[\frac{(3m_i \cdot r_i)}{r_i^2} r_i - m_i\right]$$

- **Mössbauer spectroscopy**
  - Hyperfine field B_{hf} (0) = 4.86(5) T
    =>
    Saturation moment μ **= 0.25 - 0.32** μ_B

- **Neutron scattering**
  - Saturation moment μ **= 0.36(5)** μ_B

Randomly Oriented Magnetic Moments

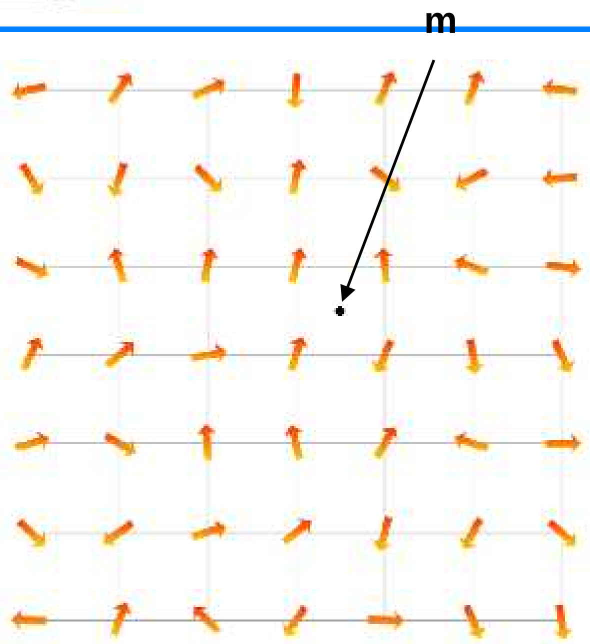
-

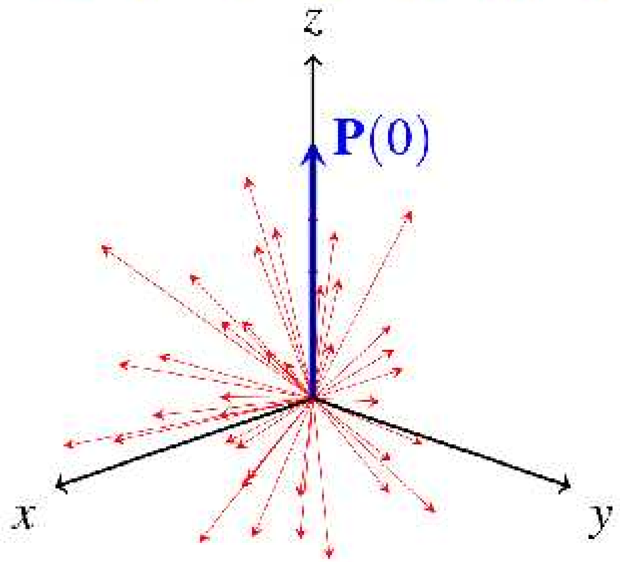
Short Range Magnetism

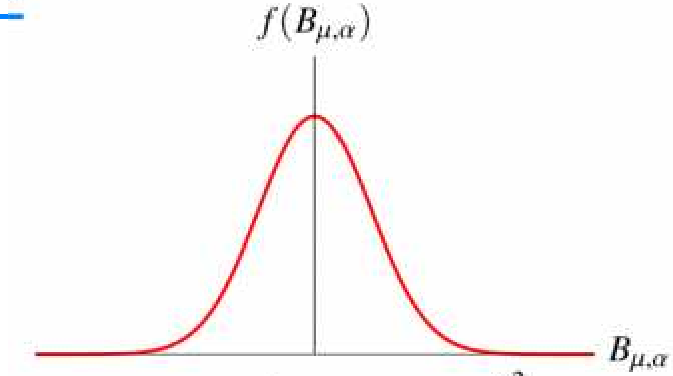
-

Magnetic Disorder

Randomly Oriented Moments







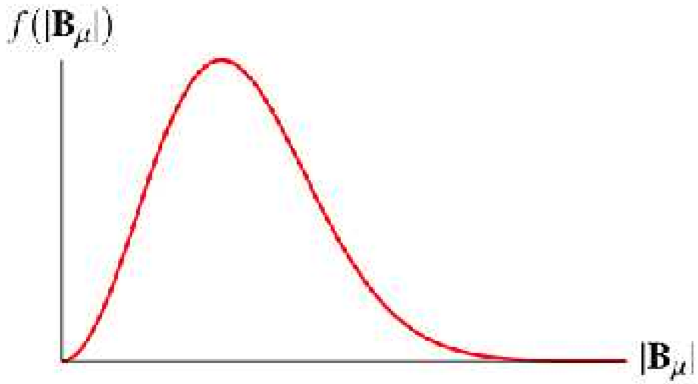
$$f(B_{\mu,\alpha}) = \frac{1}{\sqrt{2\pi\langle\Delta B_\mu^2\rangle}} \exp\left[-\frac{B_{\mu,\alpha}^2}{2\langle\Delta B_\mu{}^2\rangle}\right]$$

If isotropic:

$$f(|\mathbf{B}_\mu|) = f(\mathbf{B}_\mu)\, 4\pi B_\mu^2$$

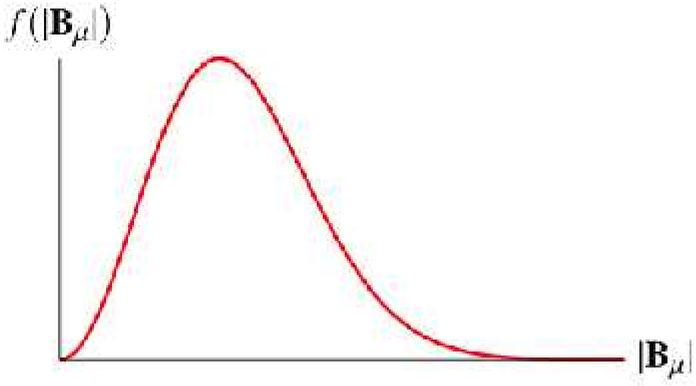
Maxwell distribution:

$$f(|\mathbf{B}_\mu|) = \frac{1}{\sqrt{2\pi\langle\Delta B_\mu^2\rangle^3}} 4\pi B_\mu^2 \exp\left[-\frac{B_\mu^2}{2\langle\Delta B_\mu^2\rangle}\right]$$



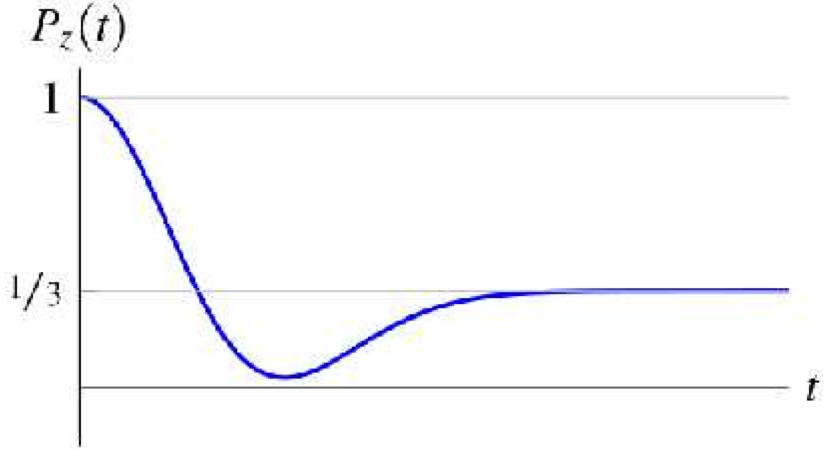
Randomly Oriented Moments

$$P_z(t) = \int f(\mathbf{B}_\mu)\left[\cos^2\theta + \sin^2\theta\cos(\gamma_\mu B_\mu t)\right]d\mathbf{B}_\mu$$



Kubo-Toyabe **function**

$$\boxed{P_z(t) = \frac{1}{3} + \frac{2}{3}\left[1 - \gamma_\mu^2\langle\Delta B_\mu^2\rangle t^2\right]\exp\left[-\frac{\gamma_\mu^2\langle\Delta B_\mu^2\rangle t^2}{2}\right]}$$

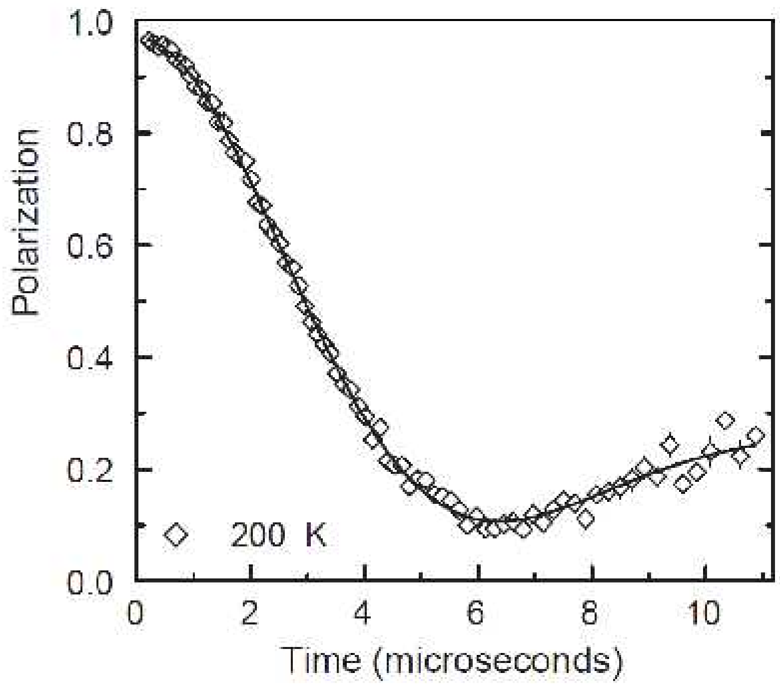


Example:
Kubo-Toyabe depolarization due to nuclear moments InN and MnSi

InN & MnSi

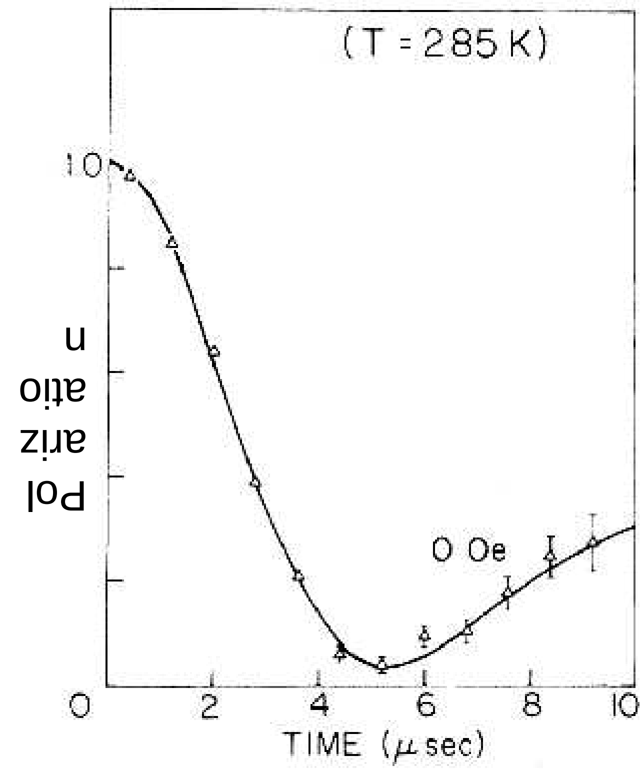
InN
Semiconductor
Study of the hydrogen-related defect chemistry

Y.G. Celebi et al., Physica B 340-342, 385 (2003)



MnSi
system lacking inversion symmetry
itinerant-electron magnet MnSi

R.S. Hayano et al., Phys. Rev. B 20, 850 (1979)



In the paramagnetic state, a KT function is very often observed reflecting the small field distribution created by the nuclear moments

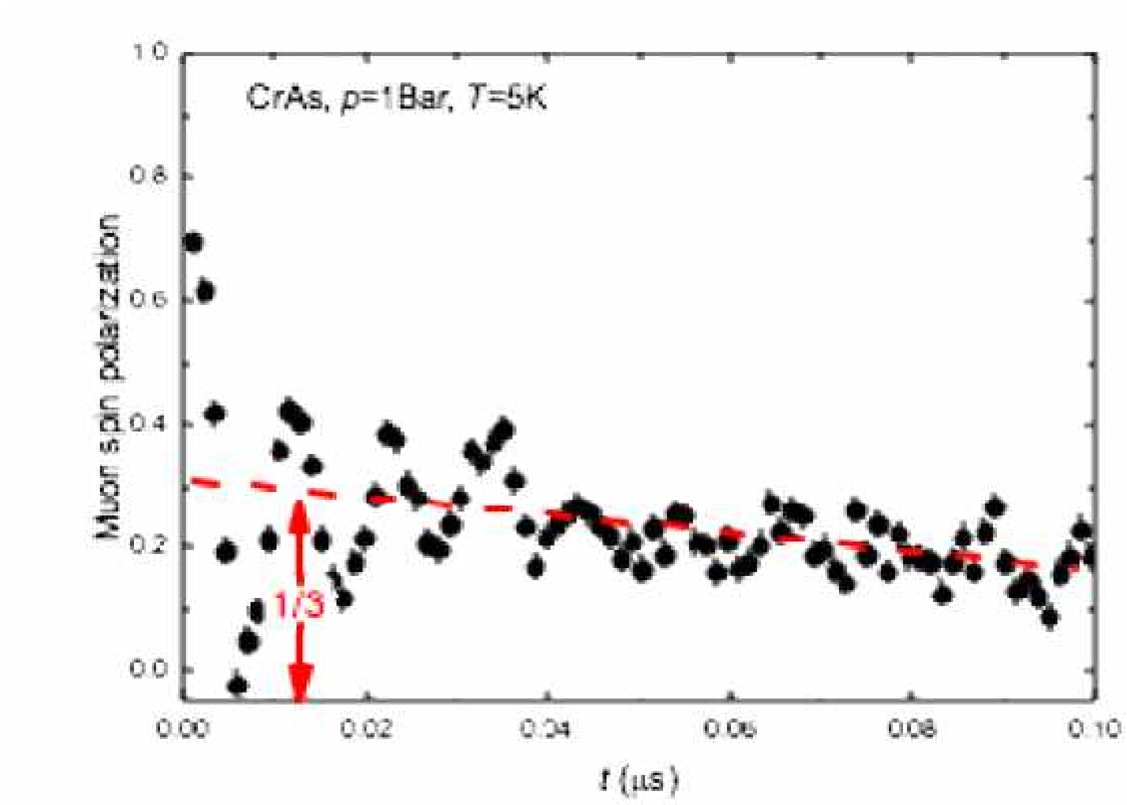
Detection of Magnetic Phase Separation

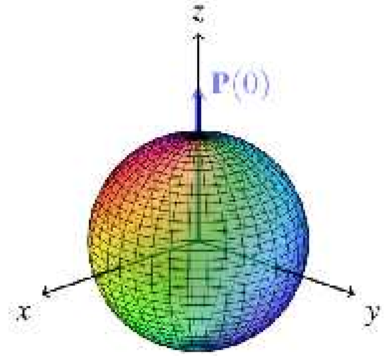
-

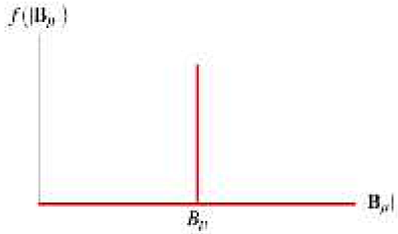
Coexistence of Different Magnetic Phases

Example:
Helical magnetic order in MnAs

Experiments in Zero Field



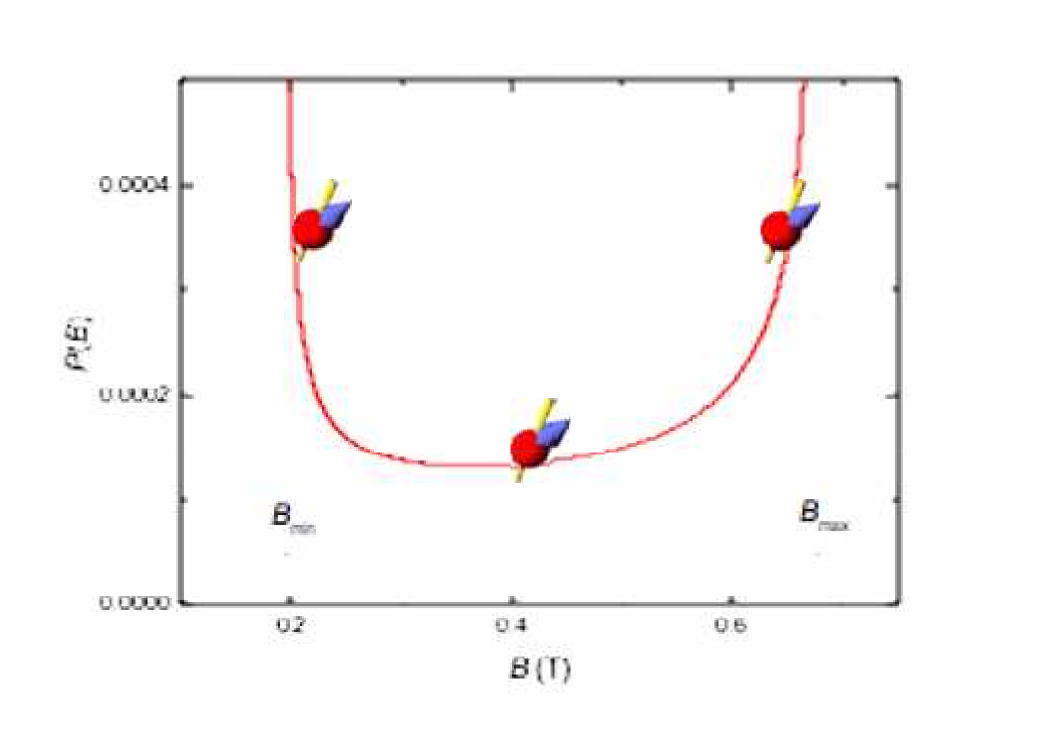




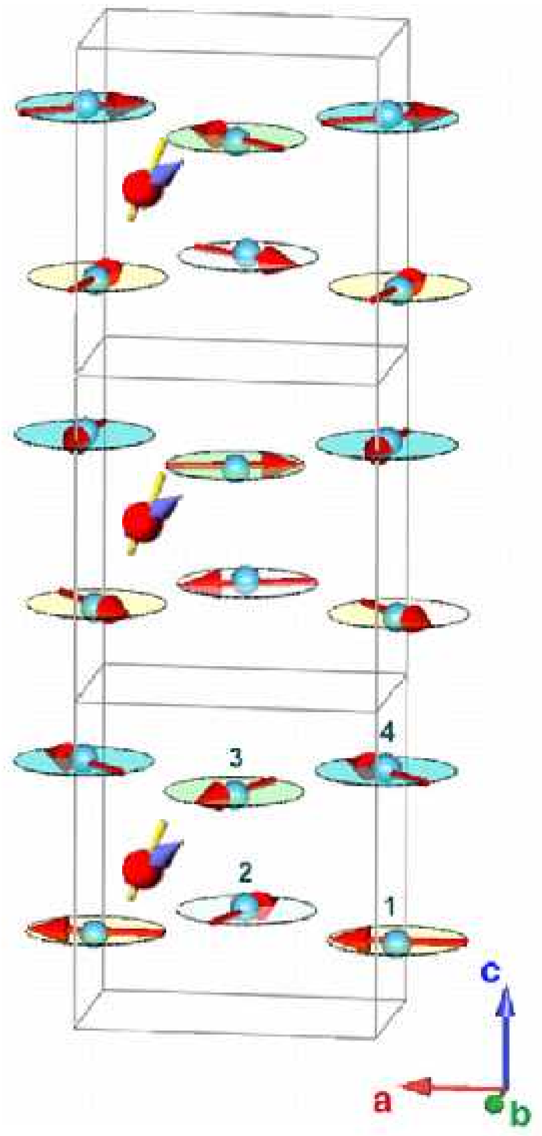
$$P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma_\mu B_\mu t)$$

At ambient pressure CrAs is 100% magnetic!

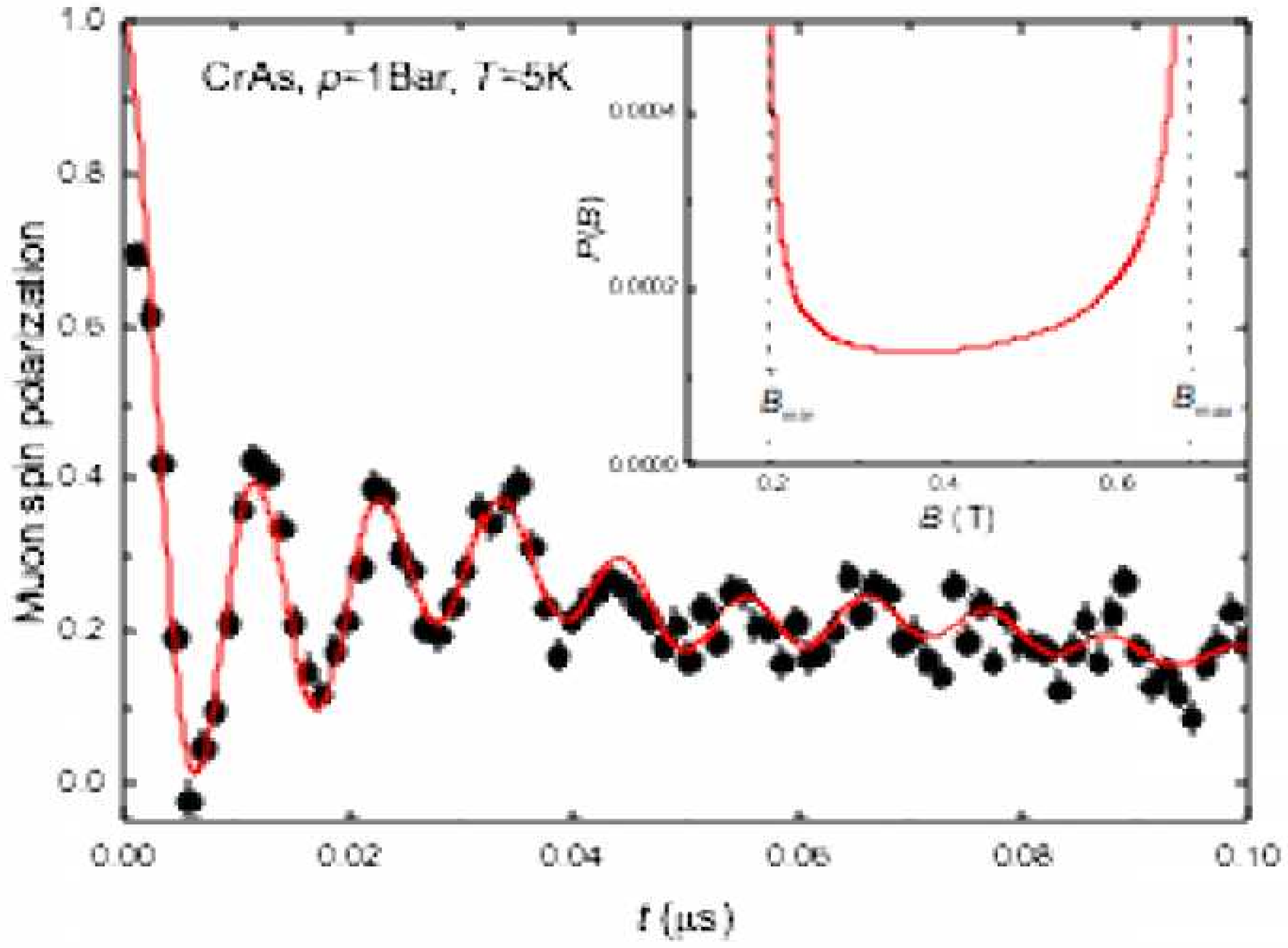
Helical magnetic order



$$P(B) = \frac{2}{\pi}\frac{B}{\sqrt{(B^2 - B_{min}^2)(B_{max}^2 - B^2)}}$$



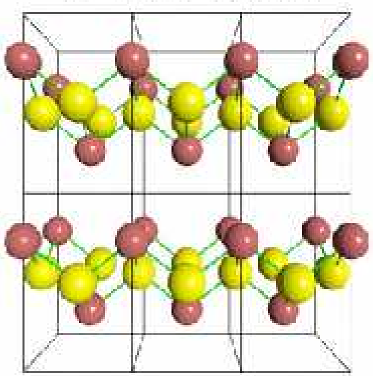
Helical magnetic order

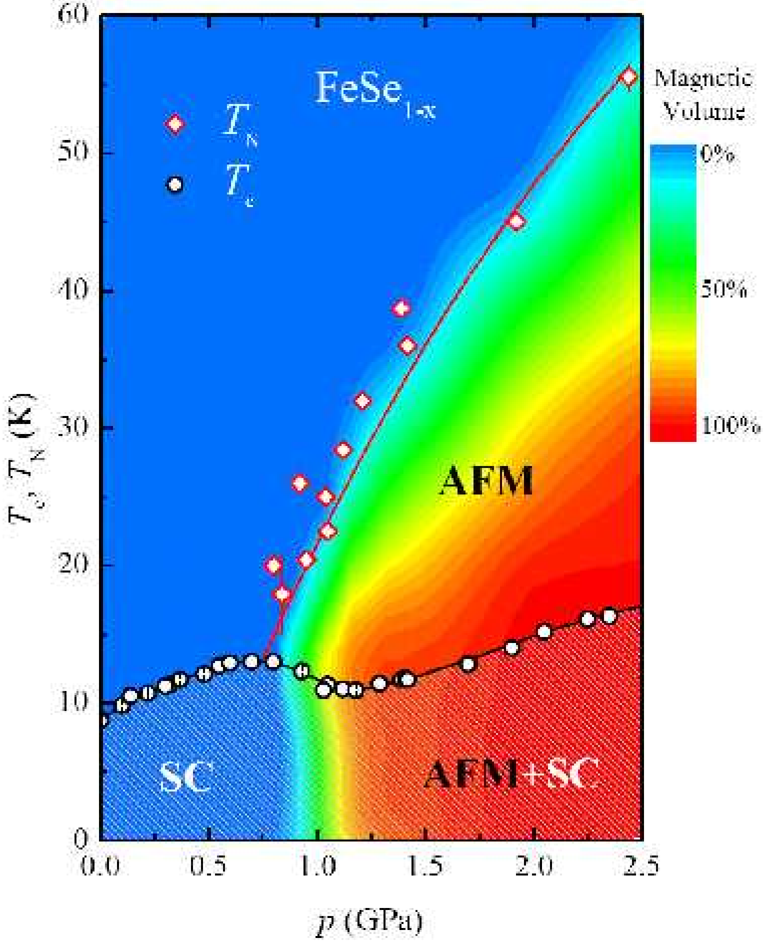


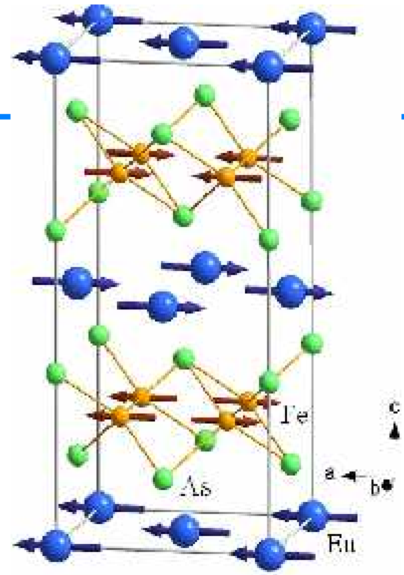
Confirmation of helical type of magnetic order in CrAs

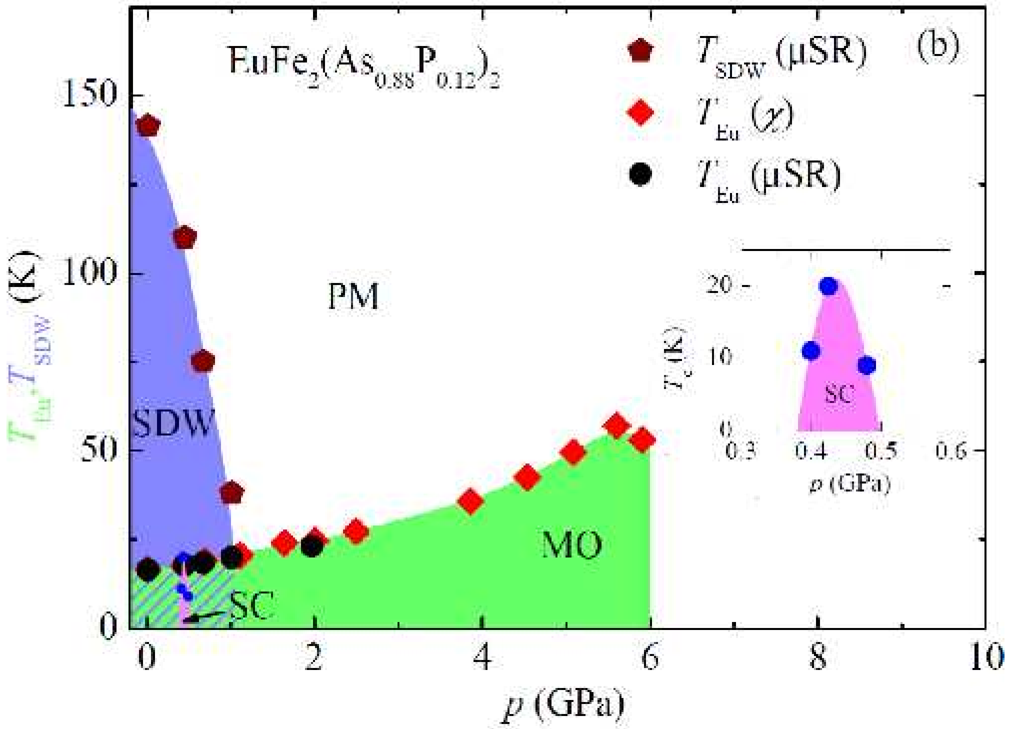
Example:
Microscopic Coexistence of Superconductivity and Magnetism in Fe-based superconductors

FeSe (11-family)



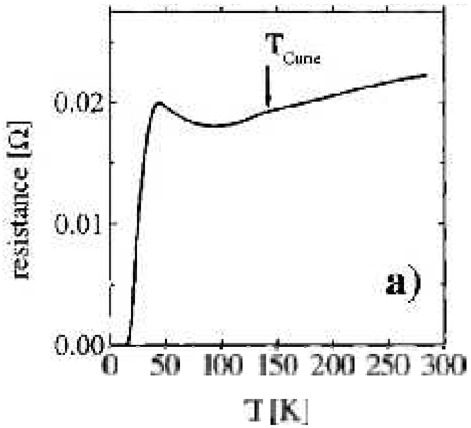




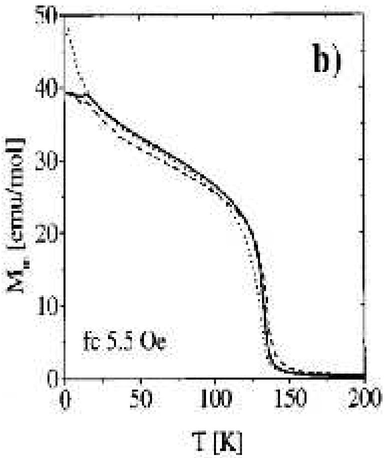


$RuSr_2GdCu_2O_8$

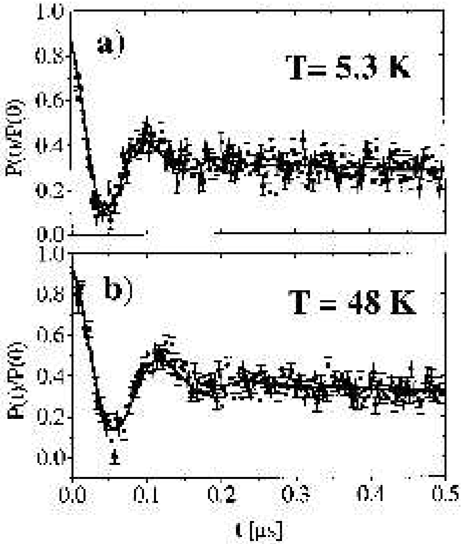
Resistivity:
(**superconductivity**)



Magnetization:
(**ferromagnetism**)



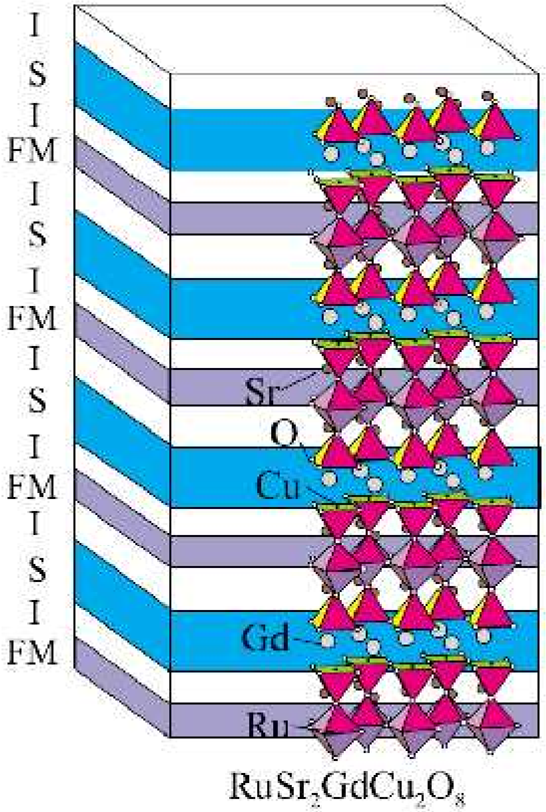
μSR:



C. Bernhard et al., Phys. Rev. B 59 (1999) 14099

~100%
Magnetic volume

Mikroscopic coexistence of superconductivity and magnetism

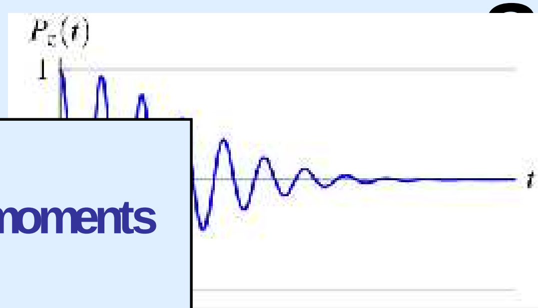


Structure:
T. Nachtrab et al., Phys. Rev. Lett. **92** (2004) 117001

Static Magnetism Probed by ZF SR

Single Crystals – Magnet

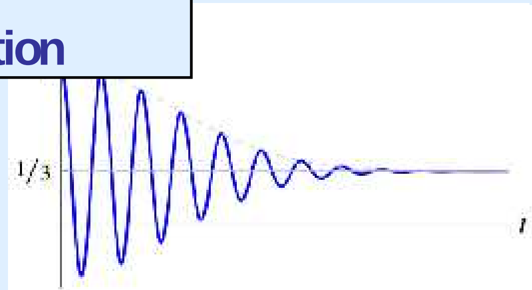
$$P_z(t) = \exp\left[-\frac{1}{2}\gamma_\mu^2\langle\Delta B_\mu{}^2\rangle t^2\right] \cos(\gamma_\mu B_\mu t)$$



- **Frequency:**
  Size of the magnetic moments
- **Damping:**
  Inhomogeneity
- **Amplitude:**
  Magnetic volume fraction

Polycrystals – Magnet

$$P_z(t) = \frac{1}{3} + \frac{2}{3}\exp\left[-\frac{1}{2}\gamma_\mu^2\langle\Delta B_\mu{}^2\rangle t^2\right] \cos\left[\gamma_\mu \langle B_\mu\rangle t\right]$$



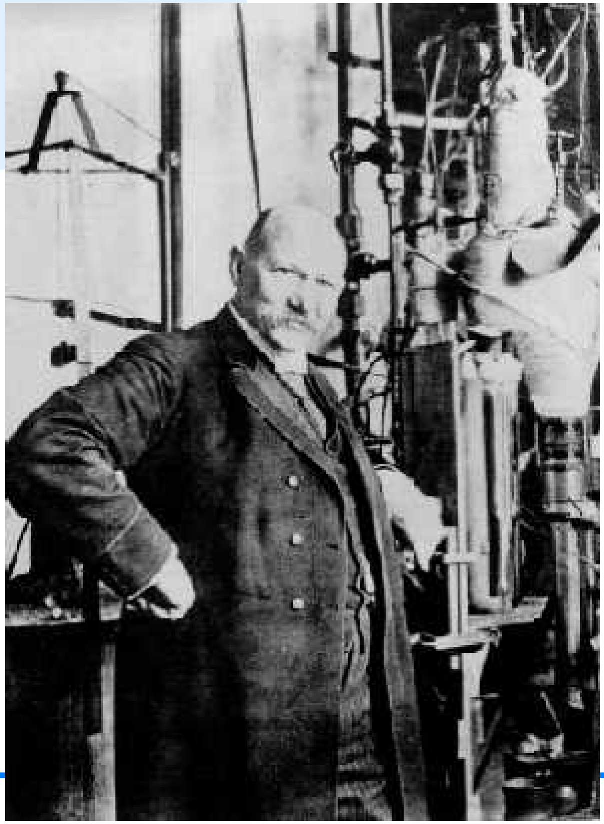
Randomly oriented static moments

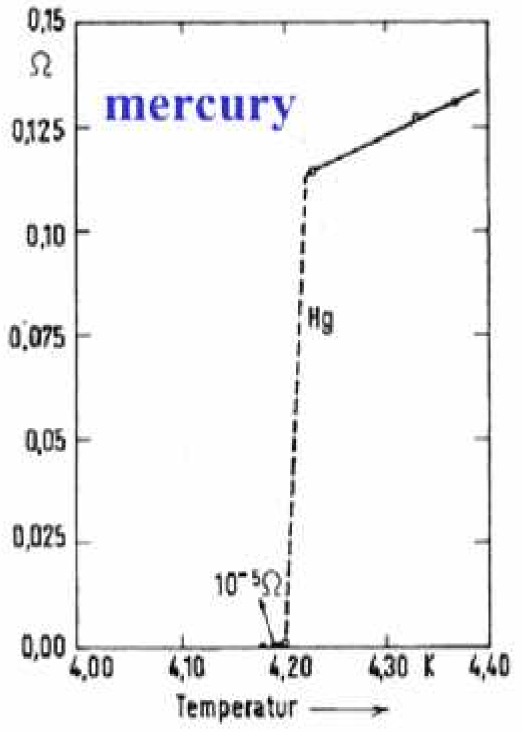
$$P_z(t) = \frac{1}{3} + \frac{2}{3}\left[1 - \gamma_\mu^2\langle\Delta B_\mu^2\rangle t^2\right] \exp\left[-\frac{\gamma_\mu^2\langle\Delta B_\mu^2\rangle t^2}{2}\right]$$

Muon Spin Rotation on Superconducting Materials

Discovery by **Kamerlingh Onnes** in 1911 in mercury

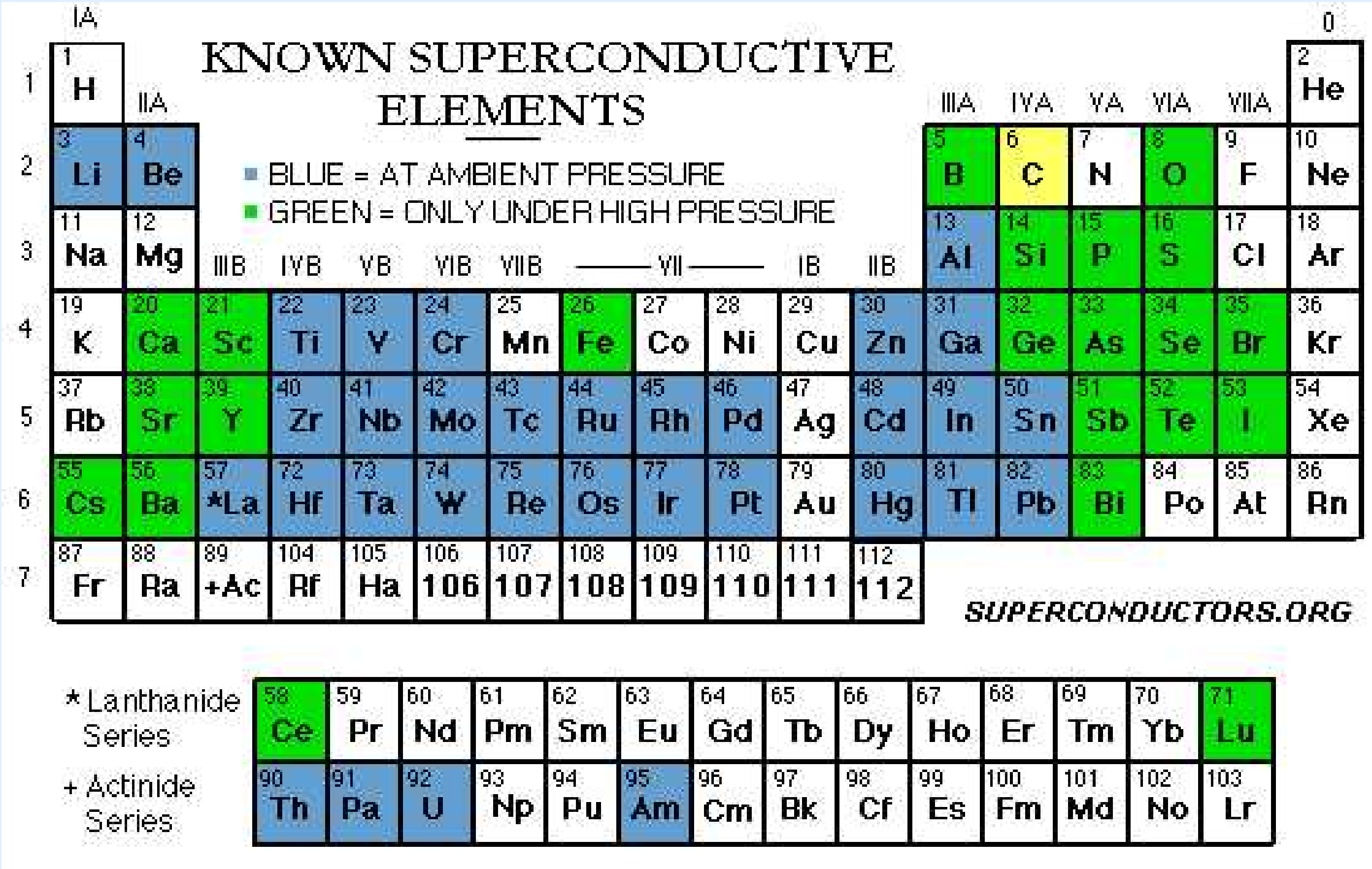
Received the Nobel Prize in 1913 for "*his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium*".





Temperature dependence of the resistance of a Hg sample

Superconductivity -- Introduction



Superconductivity -- Timeline

1908 **Kammerling Onnes**: production of liquid helium 

1911 **Kammerling Onnes**: discovery of zero resistance

1933 **Meissner and Ochsenfeld**: superconductors expell applied magnetic fields (MOE)

1935 **F. and H. London:** MOE is a consequence of the minimization of the electromagnetic free energy carried by superconducting current

1950 **Ginzburg and Landau**: phenomenological theory of superconductors 

1950 **Maxwell and Reynolds et al.**: isotope effect

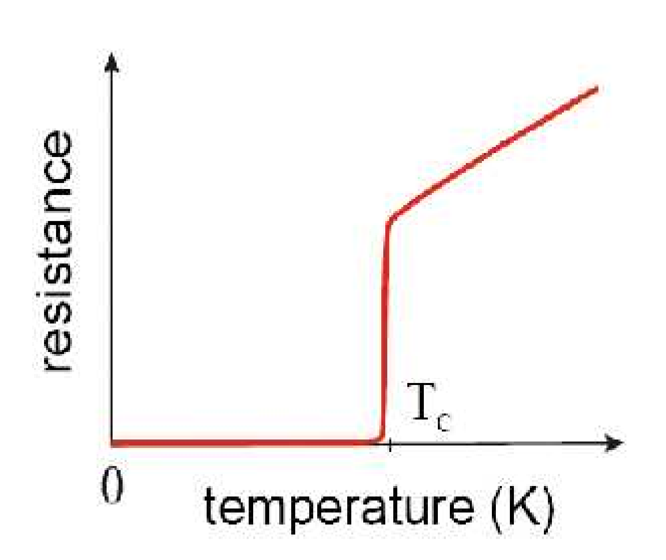
1957 **Abrikosov**: 2 types of superconductors (magnetic flux) 

1957 **Bardeen, Cooper, and Schrieffer**: BCS theory -- superconducting current as a superfluid of Cooper pairs 

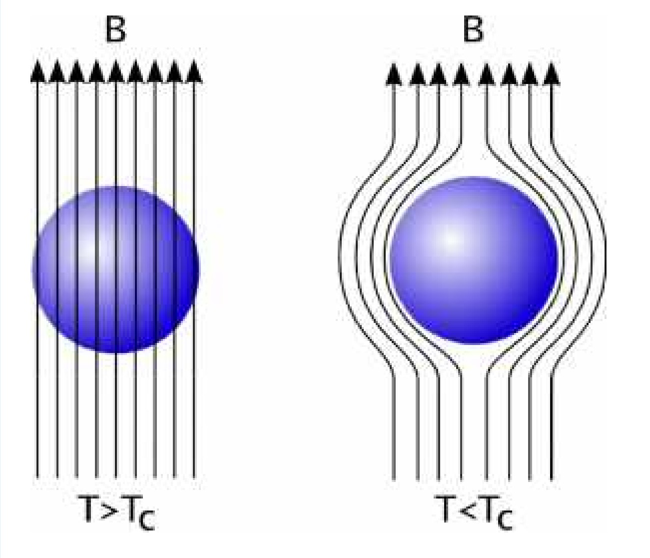
1962 **Josephson**: Joesephson effect 

1986 **Berdnorz and Müller**: High-Tc's superconductors 

Main Characteristics:



Kamerlingh Onnes

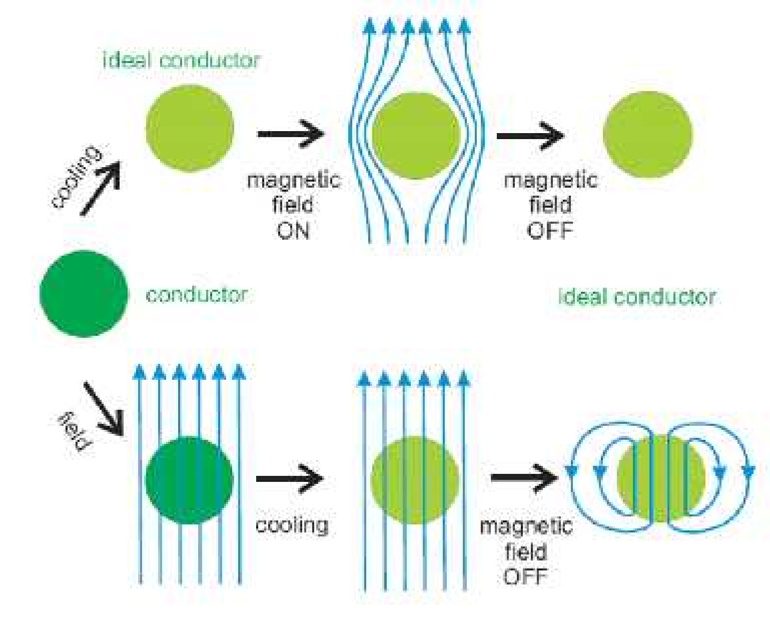


Meissner and Ochsenfeld

Is a superconductor "just" an ideal conductor?

New thermodynamic state of matter!

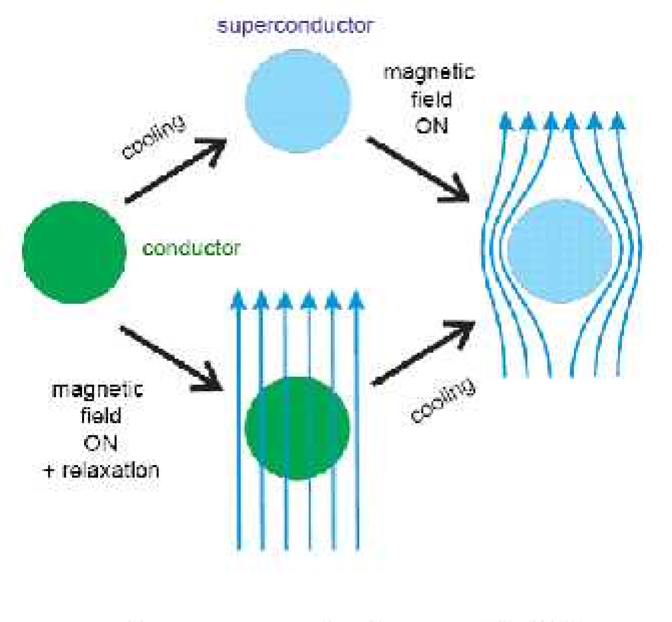
Ideal Conductor



An ideal conductor in magnetic field

Lenz-Faraday's law: currents to keep *B* constant inside of the sample

Superconductor



A superconductor in magnetic field

New thermodynamic state of matter

From: Lecture on Superconductivity, Alexey Ustinov, Uni. Erlangen, 2007

Nanometer scale parameters

Magnetic penetration depth:

Coherence length:

Phenomenological London's equations

In a sample without resitance, the electrons will feel a force:

$$\mathbf{F} = -e\mathbf{E} = m\frac{\partial\langle\mathbf{v}\rangle}{\partial t}$$

F. and H. London,
Proc. Roy. Soc. **A149**, 71 (1935)

Recalling that the current density is: $\mathbf{j} = -n_s e\langle\mathbf{v}\rangle$

one obtains the first London equation (*acceleration equation*):

$$\boxed{\Lambda\frac{\partial\mathbf{j}}{\partial t} = \mathbf{E} \quad \text{with} \quad \Lambda = \frac{m}{n_s e^2}}$$

Taking the curl of this equation

using the 3rd and 4th Maxwell equations

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{j} \qquad (\text{assuming } \frac{\partial\mathbf{E}}{\partial t} = 0)$$

$$\nabla\times\nabla\times\vec{B} = \nabla\left(\nabla\bullet\vec{B}\right) - \nabla^2\vec{B}$$

one obtains:

$$\Delta\frac{\partial\mathbf{B}}{\partial t} = \frac{1}{\lambda^2}\frac{\partial\mathbf{B}}{\partial t}$$

$$\Delta\frac{\partial\mathbf{j}}{\partial t} = \frac{1}{\lambda^2}\frac{\partial\mathbf{j}}{\partial t}$$

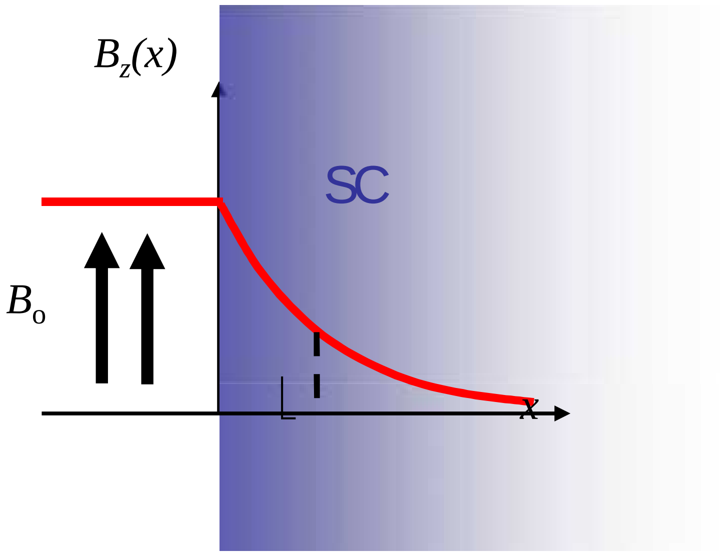
$$\text{with: } \lambda^2 = \frac{\Lambda}{\mu_0} = \frac{m}{\mu_0 e^2 n_s}$$

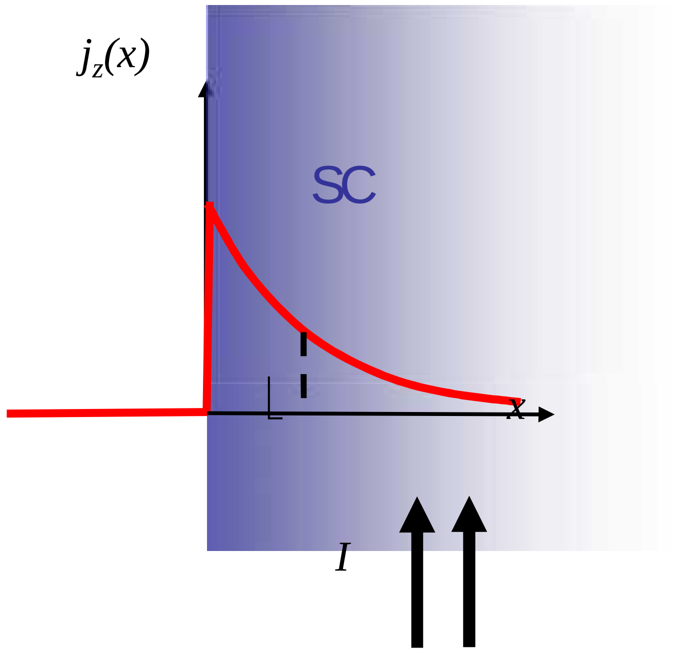
$$\boxed{\begin{aligned} \Delta\mathbf{B} &= \frac{1}{\lambda^2}\mathbf{B} \\ \Delta\mathbf{j} &= \frac{1}{\lambda^2}\mathbf{j} \\ \text{with: } \lambda^2 &= \frac{\Lambda}{\mu_0} = \frac{m}{\mu_0 e^2 n_s} \end{aligned}}$$

1st Nano-scale Param.: London Penetration Depth

$$\Delta \mathrm{B} = \frac{1}{\lambda^2}\mathrm{B}$$

$$\Delta \mathbf{j} = \frac{1}{\lambda^2}\mathbf{j}$$





$$\boxed{B_z(x) = B(0)\exp(-x/\lambda)}$$

$$\boxed{j_z(x) = \frac{I}{2\pi R\lambda}\exp(-x/\lambda)}$$

$$\boxed{\lambda = \sqrt{\frac{m^*}{\mu_0 c^2 n_s}}}$$

Ginzburg-Landau Equations (1950)

Powerful phenomenological theory, based on the Landau theory of second order transition.

Pseudowave-function ψ acting as order parameter (in the normal phase $= 0$, in the superconducting phase $\neq 0$).

ψ describes the superconducting electrons and their density

$$n_s = |\psi(\mathbf{r})|^2$$

The free energy density f_s can be expanded in a series:

$$\boxed{f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - 2e\mathbf{A})\psi|^2 + \frac{|\mathbf{B}|^2}{2\mu_0}}$$

The order parameter and the vector potential are obtained by minimizing the Ginzburg-Landau formula with respect to ψ and $\mathbf{A}$.

2^{nd} Nano-scale Param.: Coherence Length

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - \cancel{2eA})\psi|^2 + \cancel{\frac{|B|^2}{2\mu_0}}$$

Let assume a situation without field and at an interface

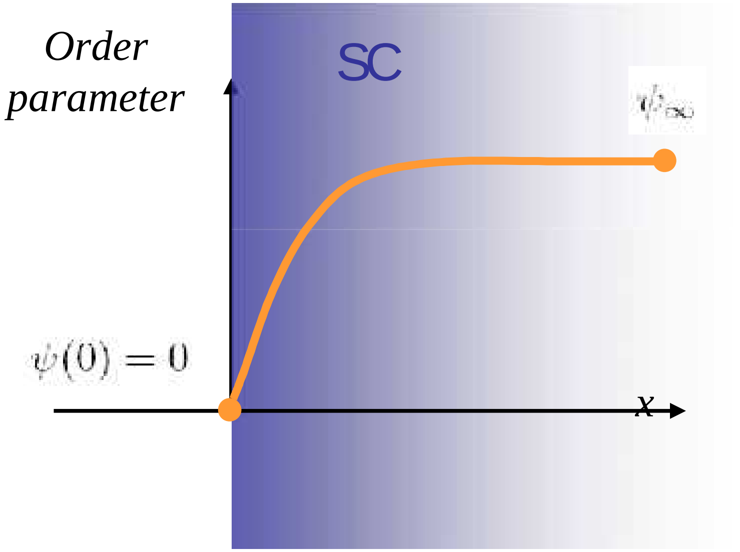
vaccuum/superconductor.

Minimizing the free energy with respect to ψ

$$\alpha\psi + \beta|\psi|^2\psi - \frac{\hbar^2}{2m}\nabla^2\psi = 0$$

Taking into account that $\psi(0) = 0$ and that $\psi(x \gg 0) = \psi_\infty$

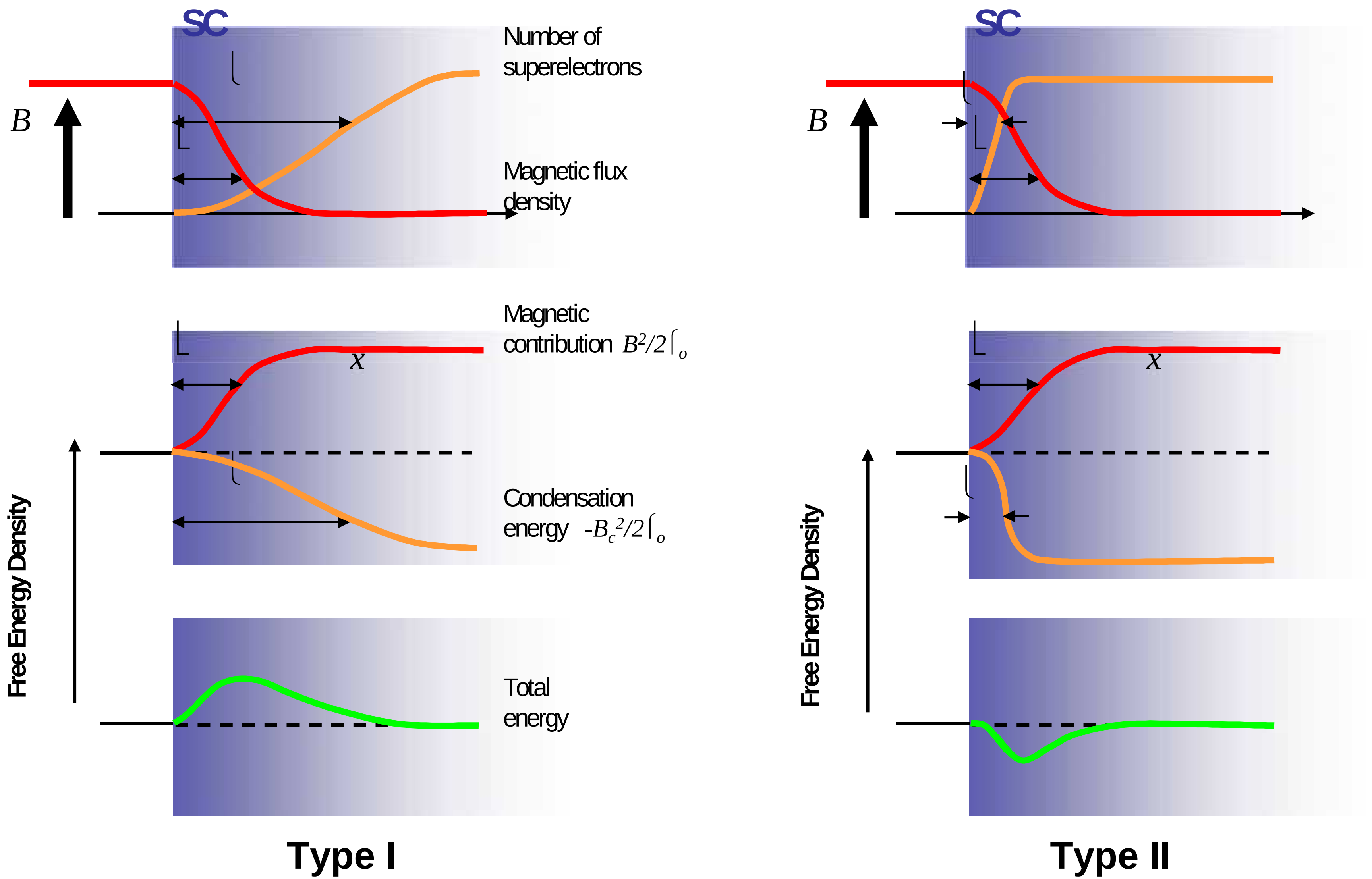
$$\psi(x) = \psi_\infty \tanh\left(\frac{x}{\sqrt{2}\,\xi}\right)$$



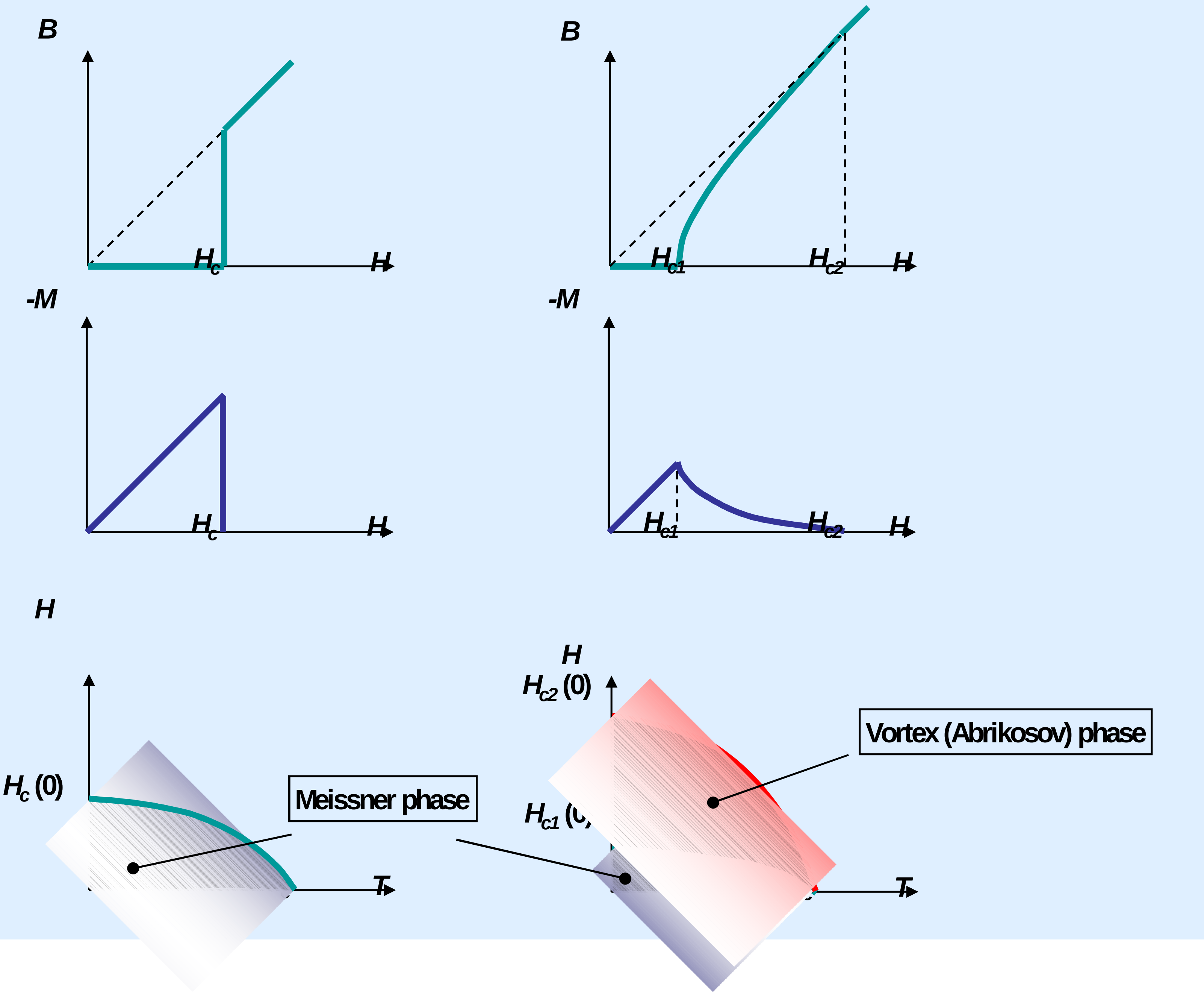
$$\boxed{\text{with: } \xi = \sqrt{\frac{\hbar^2}{2m|\alpha|}}} \qquad \psi_\infty^2 = -\frac{\alpha}{\beta}$$

$$\boxed{\xi = \frac{\hbar v_F}{\pi\Delta}}$$

Type I and Type II Superconductors

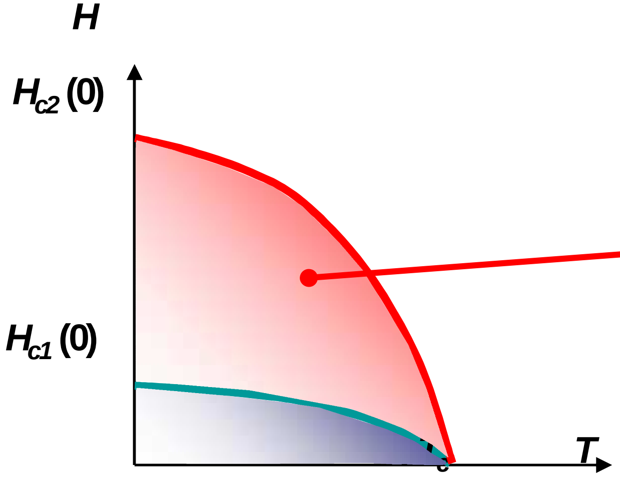


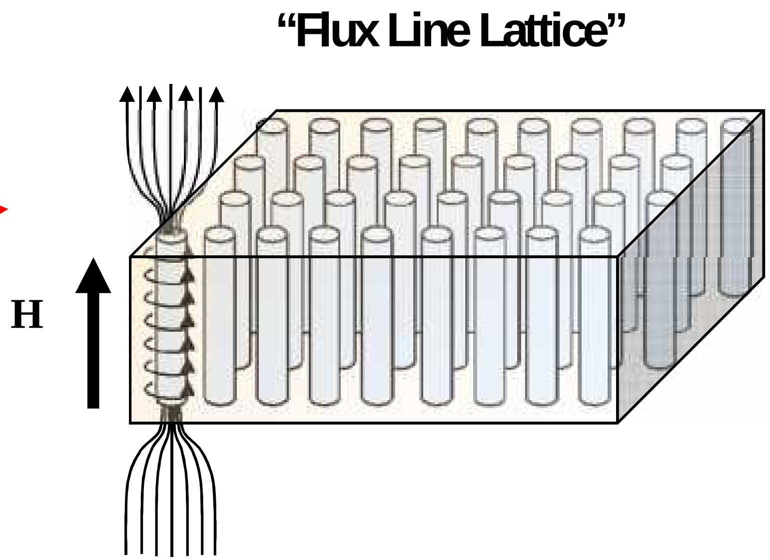
Type I (⎣ < ⎩) Ty



B
H_c
H
-M
H_c
H
H
H_c (0)
T
Meissner phase
B
H_c1
H_c2
H
-M
H_c1
H_c2
H
H
H_c2 (0)
H_c1 (0)
T
Vortex (Abrikosov) phase

Vortex (Abrikosov) phase

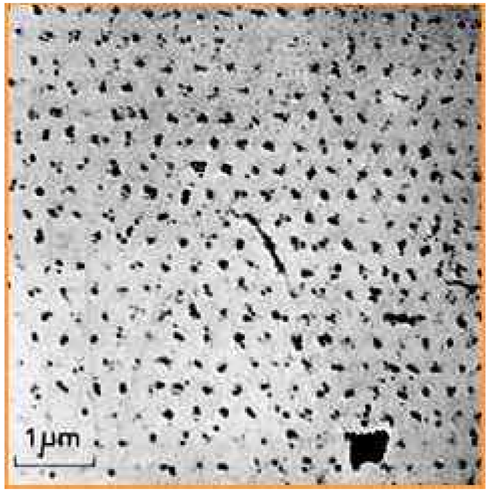




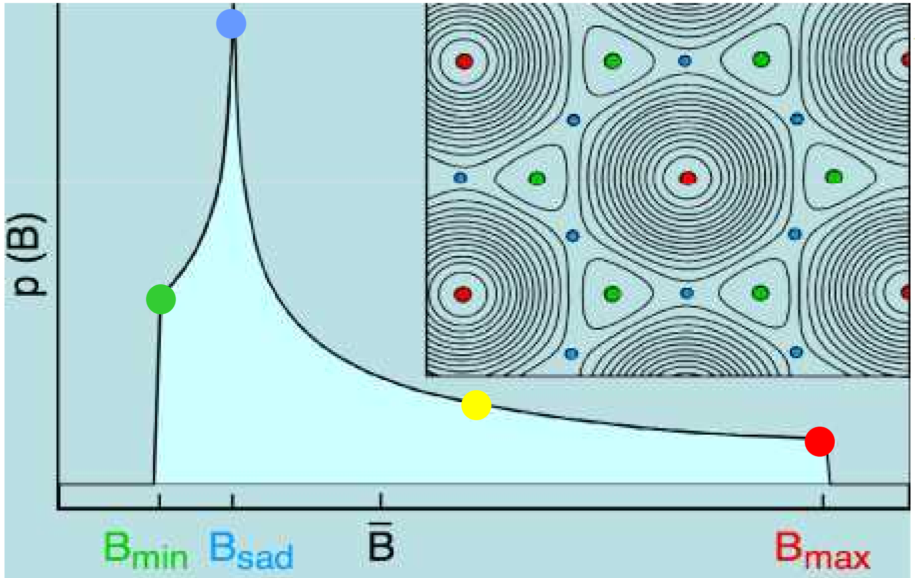
Magnetic flux quantum

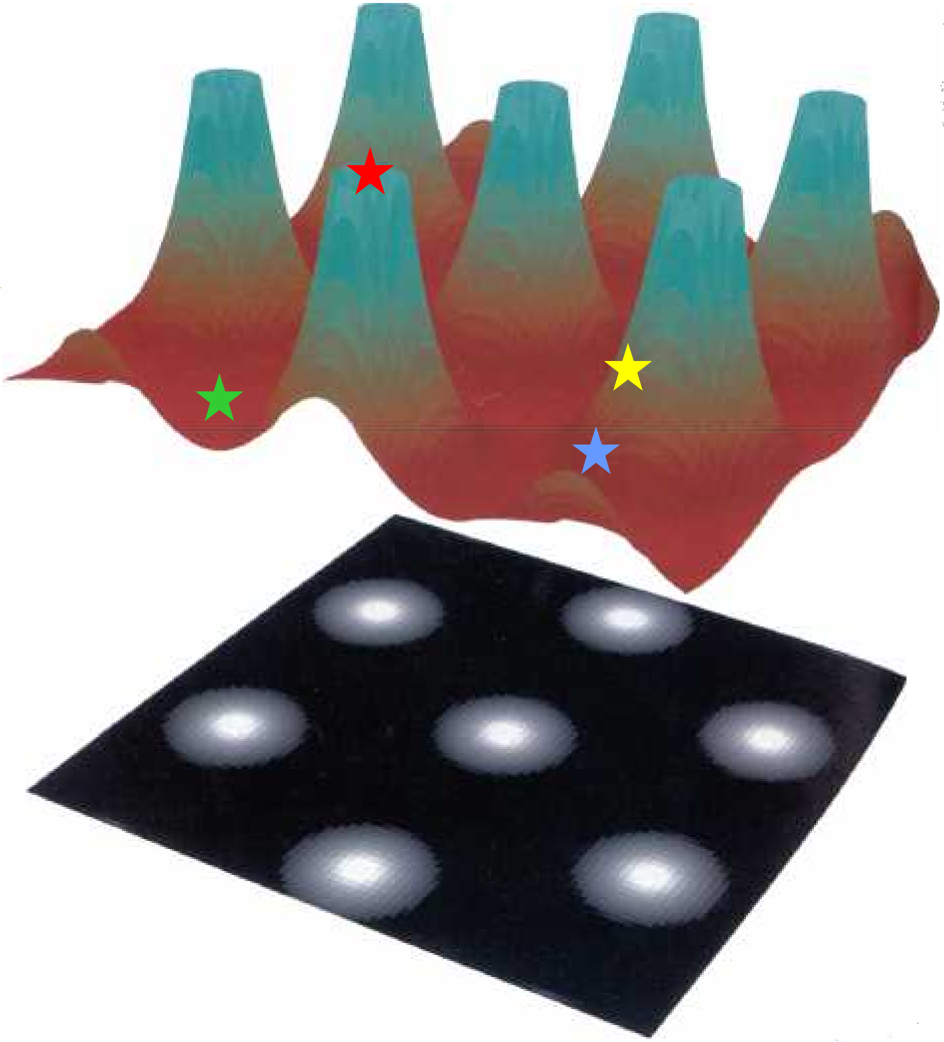
$\Phi_0 = h/2e = 2.067 \cdot 10^{-15}$ Wb

Bitter Decoration
Pb-4at%In rod, 1.1K, 195G
U. Essmann and H. Trauble, Phys. Lett 24A, 526 (1967)



Muons and Field Distribution in Type II S.C.

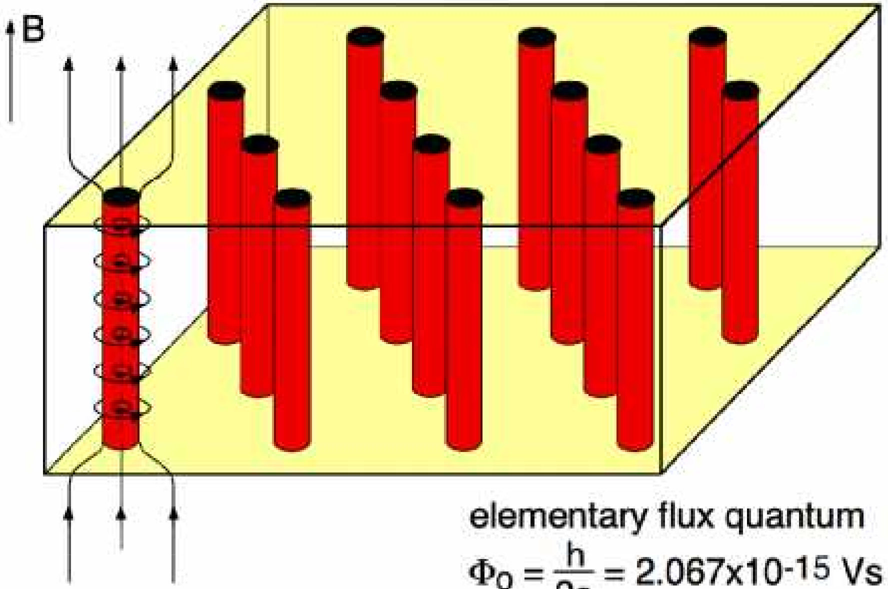




Bishop et al., Scientific American ***48*** *(1993)*

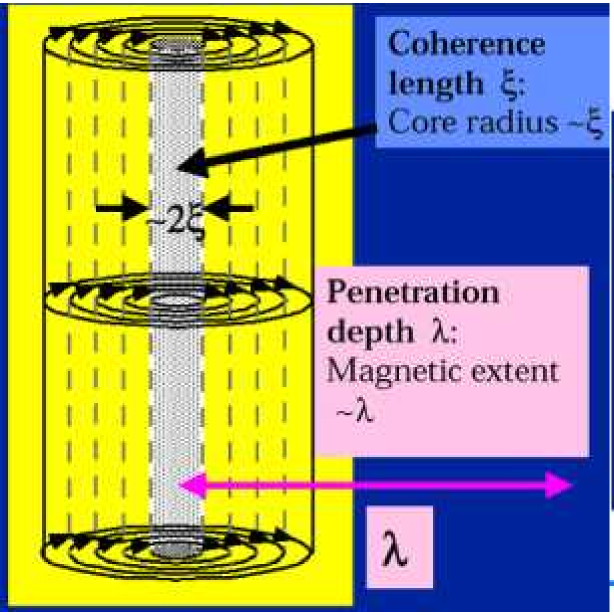
Using Bulk Muon Spin Rotation to Study Superconducting Materials

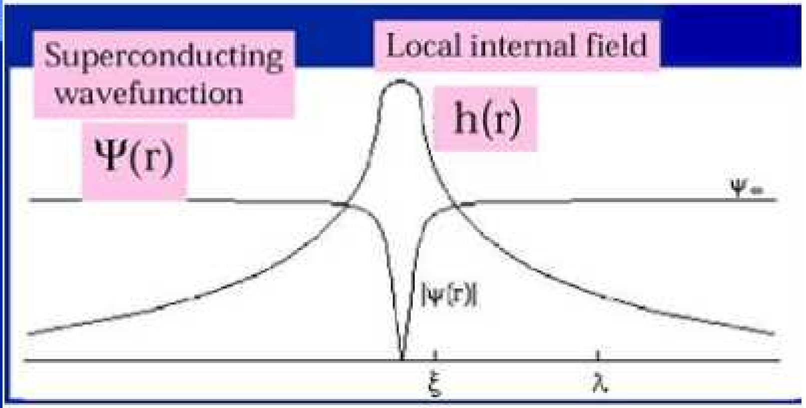
Flux-line lattice (Abrikosov lattice)



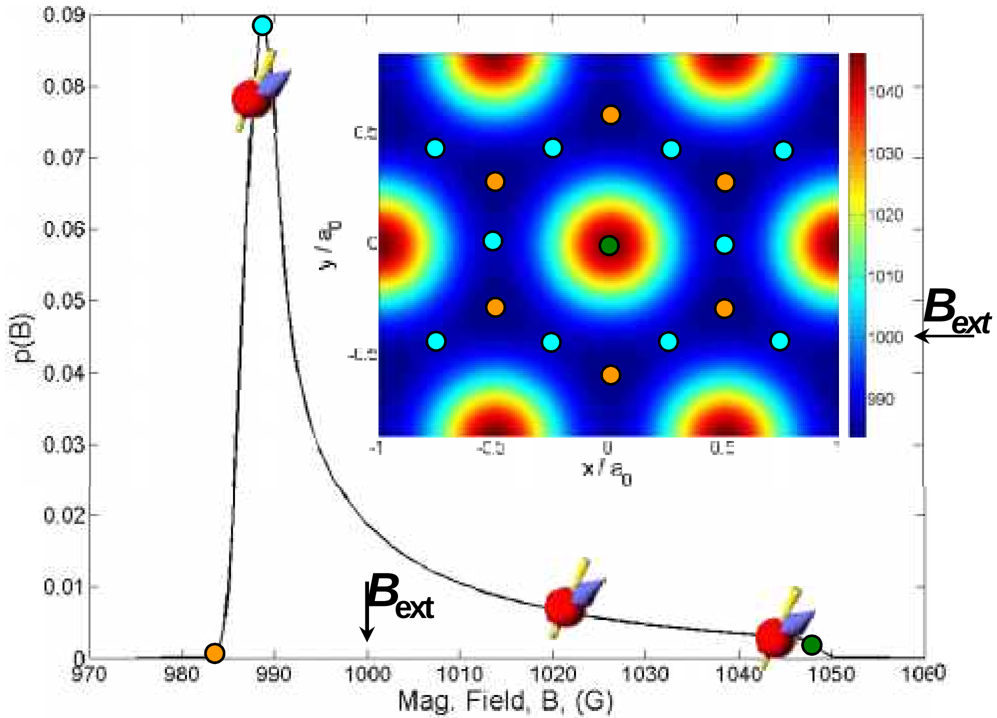
elementary flux quantum

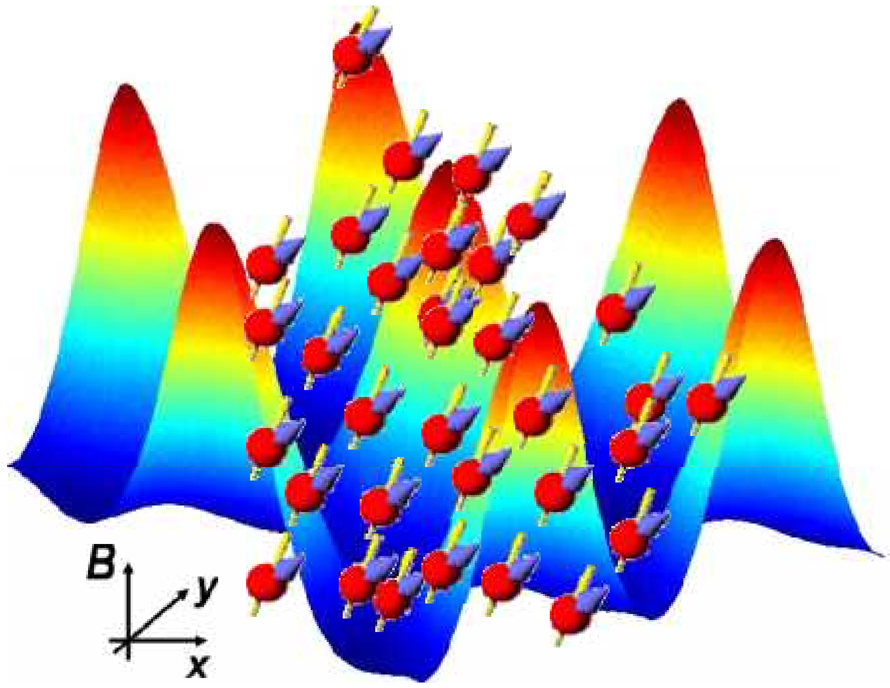
$$\Phi_0 = \frac{h}{2e} = 2.067\text{x}10^{-15}\ \text{Vs}$$





Field Distribution in a Type II S.C.



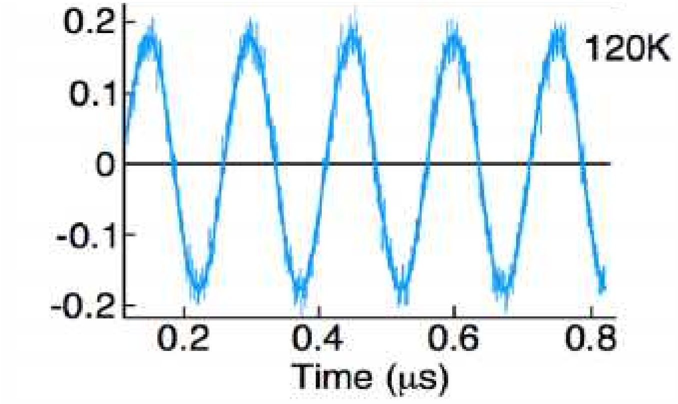


Since the muon is a local probe, the μSR relaxation function is given by the weighted sum of all oscillations:

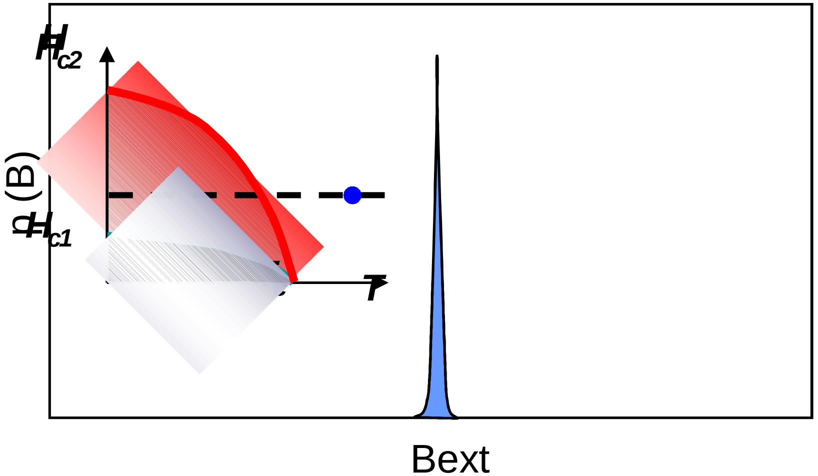
$$G(t) = \int f(\mathbf{B}_\mu)\cos(\gamma_\mu B_\mu t)\,d\mathbf{B}_\mu$$

μSR a Type II Superconductor

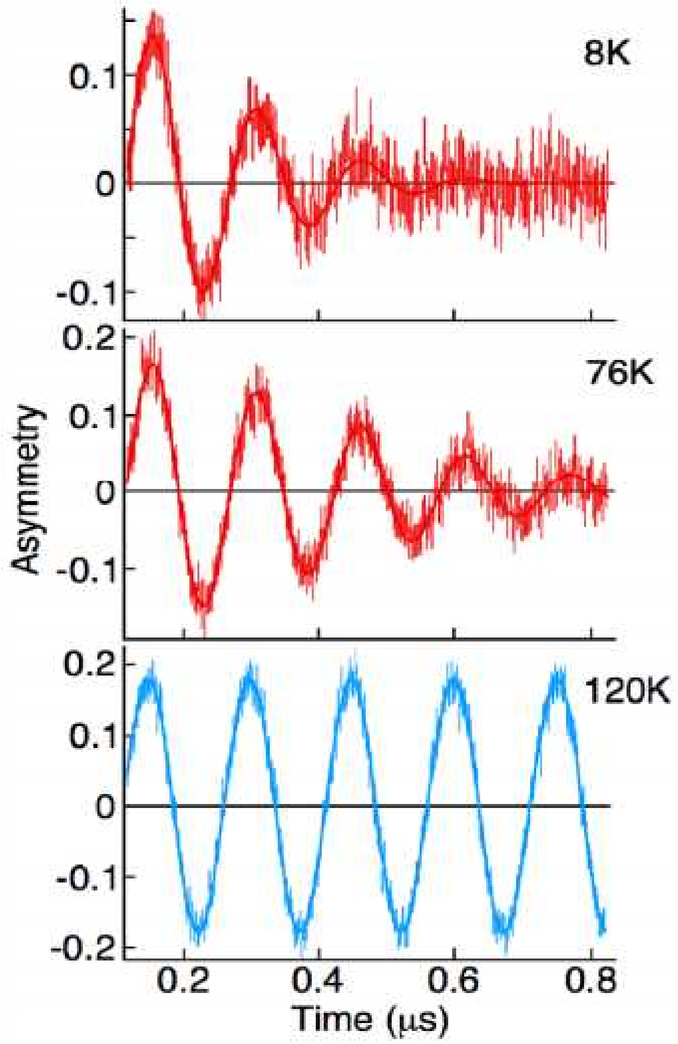
$T > T_c$



$YBa_2Cu_3O_{6.97}$
*Pümpin et al., Phys. Rev. B **42**, 8019 (1990)*

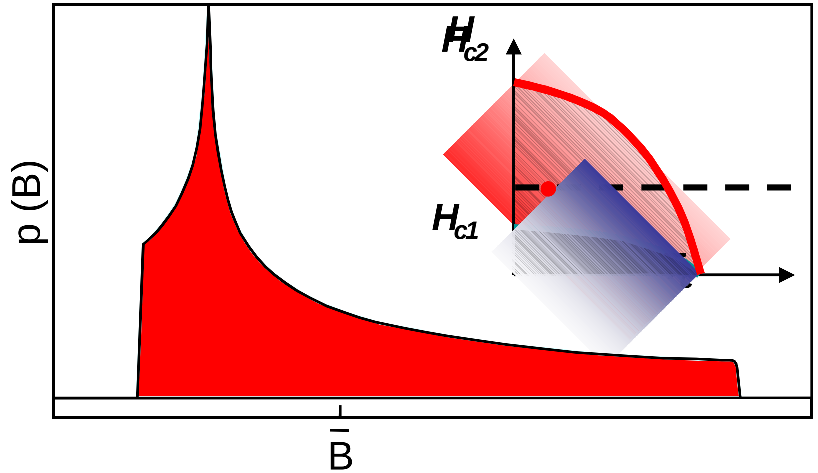


μSR a Type II Superconductor

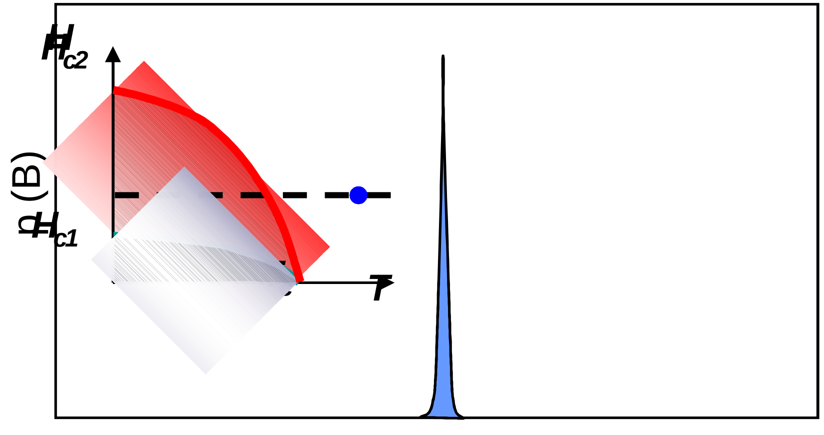


$YBa_2Cu_3O_{6.97}$
Pümpin et al., Phys. Rev. B ***42****, 8019 (1990)*

$T < T_c$



$T > T_c$



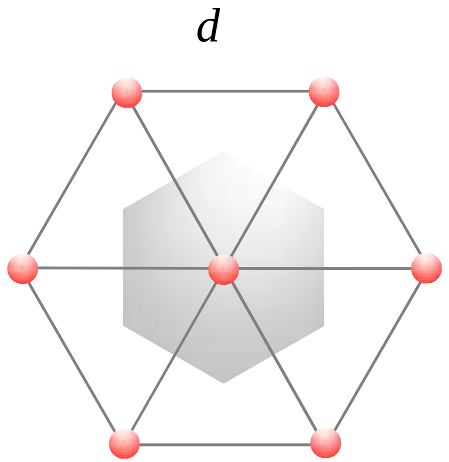
Field Distribution in "Extreme" Type II S.C.

- Ginburg-Landau parameter $\kappa \equiv \frac{\lambda}{\xi} \gg 1$
- Large range of fields (up to $B_{c2}/4$) where London model applies
- Vortex cores well separated and do not interact
- Vortex fields superimpose linearly

$$S = d^2 \frac{\sqrt{3}}{2}$$

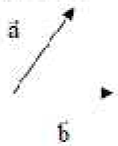
$$\phi_0 = S \cdot B$$

$$\Rightarrow d = \sqrt{\frac{2\phi_0}{B\sqrt{3}}}$$



In an ideal vortex state the vectors $\vec{r}_n$ form a periodic two dimensional lattice. Therefore [7-4] can be solved in Fourier space ($\vec{k}$ space).

For an hexagonal lattice:



$|\vec{a}| = |\vec{b}| = d,\ \vec{a}\cdot\vec{b} = \cos 60^\circ$

Reciprocal vectors:

$$\vec{a}^* = 2\pi \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot(\vec{b}\times\vec{c})} \qquad \vec{b}^* = 2\pi \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot(\vec{b}\times\vec{c})}$$

$$|\vec{a}^*| = 2\pi \frac{|\vec{b}|}{|\vec{a}\cdot\vec{b}|} = \frac{4\pi}{\sqrt{3}d} = |\vec{b}^*|$$

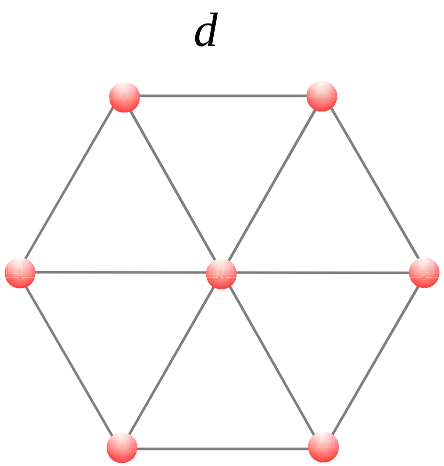
$$\vec{k}_{m,n} = m\vec{a}^* + n\vec{b}^*$$

(also hexagonal symmetry)

$$\vec{B}(\vec{r}) = \sum_{k} b_k e^{i\vec{k}\vec{r}} \qquad [7\text{-}5]$$

With Fourier components:

$$b_k = \frac{1}{S}\int B(\vec{r}) e^{-i\vec{k}\vec{r}} d^2\vec{r}$$



Field distribution: $\mathbf{B}(\mathbf{r})$?

$\mathbf{B}(\mathbf{r})$ must fulfill the modified London equation:

$$\boxed{\mathbf{B}(\mathbf{r}) - \lambda^2 \Delta \mathbf{B}(\mathbf{r}) = \phi_0 \sum_n \delta(\mathbf{r} - \mathbf{r}_n)\hat{z}}$$

We expect a periodic magnetic field

and therefore can use:

$$\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{K}} \mathbf{B}(\mathbf{K}) \exp(i\mathbf{K}\mathbf{r})$$

with Fourier components:

$$\mathbf{B}(\mathbf{K}) = \frac{1}{S} \int \mathbf{B}(\mathbf{r}) \exp(-i\mathbf{K}\mathbf{r}) d^2\mathbf{r}$$

The modified London equation becomes

(fields only along $\hat{z}$):

$$\boxed{\sum_{\mathbf{K}} (\mathbf{B}(\mathbf{K}) + \lambda^2 K^2 \mathbf{B}(\mathbf{K})) \exp(i\mathbf{K}\mathbf{r}) = N\phi_0 \sum_{\mathbf{K}} \exp(i\mathbf{K}\mathbf{r})}$$

and one finds:

$$B_z(\mathbf{K}) = \frac{B}{1 + \lambda^2 K^2}$$

With: $B_z(\mathbf{r}) = \sum_{\mathbf{K}} \frac{B}{1+\lambda^2 K^2} \exp(i\mathbf{K}\mathbf{r})$

The second moment $\langle \Delta B_z^2 \rangle = \langle B_z^2 \rangle - \langle B_z \rangle^2$

of the field distribution is given by:

$$\langle \Delta B_z^2 \rangle = \sum_{\mathbf{K} \neq 0} |B_z(\mathbf{K})|^2$$

Taking into account the perfect triangular lattice where:

$$K^2 = K_{m,n}^2 = \frac{16\pi^2}{3d^2}(m^2 + mn + n^2) \quad \text{and that} \quad K^2\lambda^2 \gg 1$$

$$< \Delta B_z^{\ 2} > = \frac{3\Phi_0^{\ 2}}{64\pi^4\lambda^4} \sum_{(m,n)\neq(0,0)} \frac{1}{(m^2 - mn + n^2)^2}$$

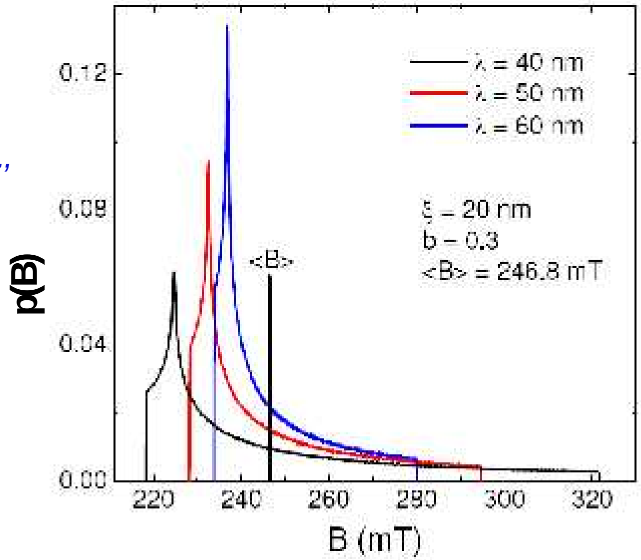
$$\boxed{\langle \Delta B_z^2 \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4}}$$

- By measuring the second moment of the field distribution (for example by µSR), we directly determine the London penetration

Extract Information from the μSR data

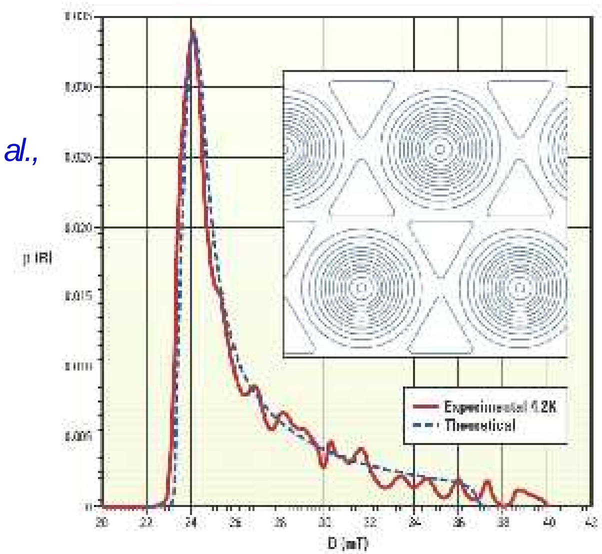
Theory

Maisuradze et al., (2008)



Pd-In alloy

*Charalambous et al., Phys. Rev. B **66**, 054506 (2002)*



Field distribution depends on the microscopic parameters of superconductivity λ and ξ

Theoretical models of the flux line lattice to fit the μSR data

▼

Structure, symmetry of the flux line lattice

Vortex motion

Characteristic lengths: magnetic penetration depth λ, radius of the vortex core - coherence length ξ

Classification scheme of superconductors

Extract Information from t[illegible] data

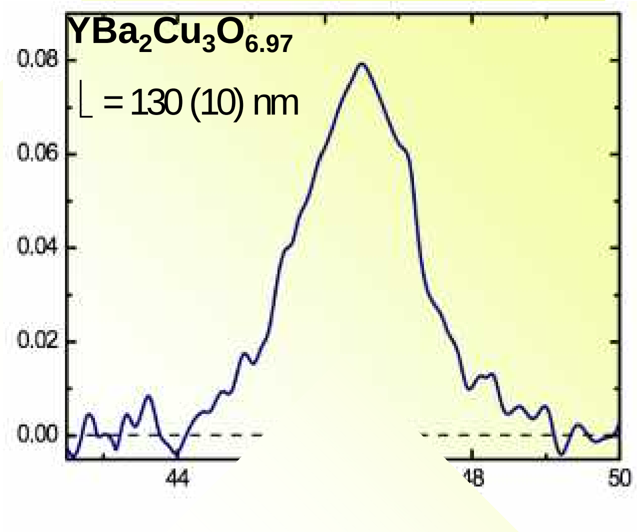
Single crystals

- Anisotropy field distribution

Polycrystals or sintered sample

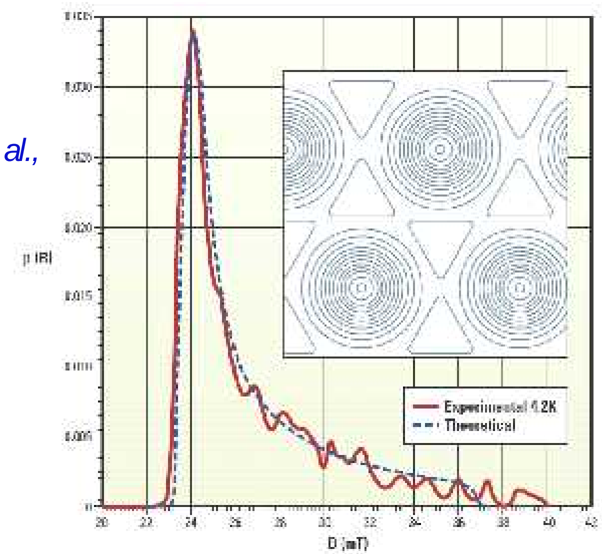
- Disorder of pinning sites
- Strong smearing of the field distribution

*Pümpin et al., Phys. Rev. B **42** 8019 (1990)*

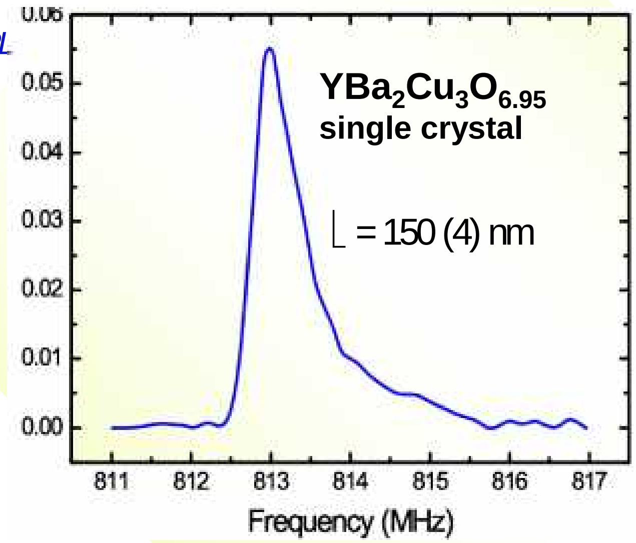


Pd-In alloy

*Charalambous et al., Phys. Rev. B **66**, 054506 (2002)*



*Sonier et al., PRL **83**, 4156 (1999)*

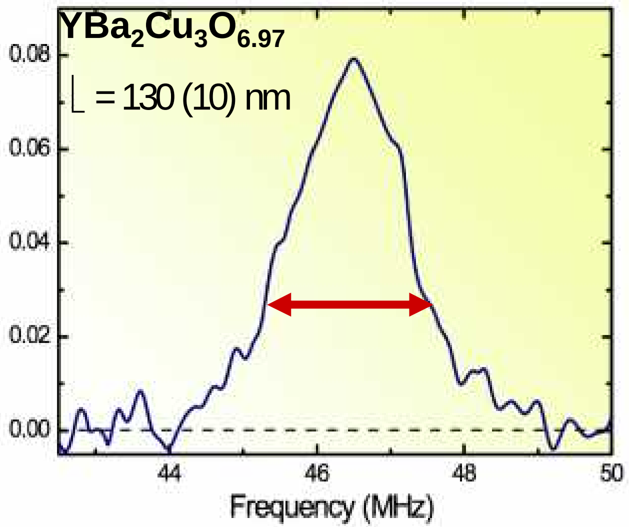


Extract Information from the μSR data

$$\underbrace{G(t) = \exp(-\frac{1}{2}\sigma^2 t^2)}_{\text{depolarization}} \times \underbrace{\cos(\gamma_\mu \langle B_\mu^z \rangle t)}_{\text{oscillations}}$$

where: $\sigma^2 = \gamma_\mu^2 \langle \Delta B_\mu^{z\,2} \rangle$

Pümpin et al., Phys. Rev. B ***42*** *8019 (1990)*



Ginzburg-Landau model

$$\langle \Delta B_z^2 \rangle = 0.00371 \frac{\phi_0^2}{\lambda^4}$$

A μSR measurement of the second moment of the field distribution allows to determine the London penetration depth λ.

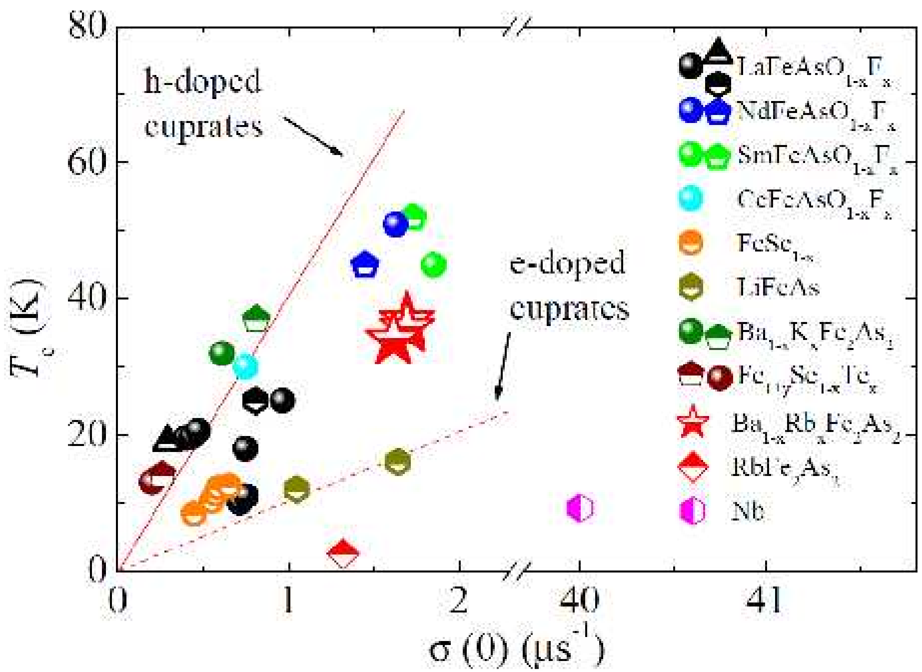
London model

$$\lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}}$$

$$\Rightarrow \quad \sigma \propto \frac{1}{\lambda^2} \propto \frac{\mu_0 e^2}{m} n_s$$

The damping of a TF-μSR spectrum is proportional to the super fluid density n_s (number of Cooper pairs).

Classification of SCs: Uemura Plot



$$T_c \text{ versus } T_F \propto \sigma \propto \frac{1}{\lambda^2} \propto \frac{\mu_0 e^2}{m} n_s$$

- **Conventional superconductors have low T_C and high superfluid density**

- **Unconventional superconductors have relatively low superfluid density ("dilute superfluid")**

Z. Guguchia, et al., Phys. Rev. B 84, 094513 (2011)

Pairing Symmetry in Cuprates

Wave function of two electrons:

$$\Psi(\mathbf{r}_1, s_1; \mathbf{r}_2, s_2) = \psi(\mathbf{r}_1, \mathbf{r}_2)\chi(s_1, s_2)$$

where:

$\psi(\mathbf{r}_1, \mathbf{r}_2)$: space part

$\chi(s_1, s_2)$: spin part

spin singlet: $\left(\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\right)$

$\Rightarrow S = 0 \rightarrow$ space part must be even.

$\Rightarrow$ s-wave (l = 0), d-wave (l = 2) etc...

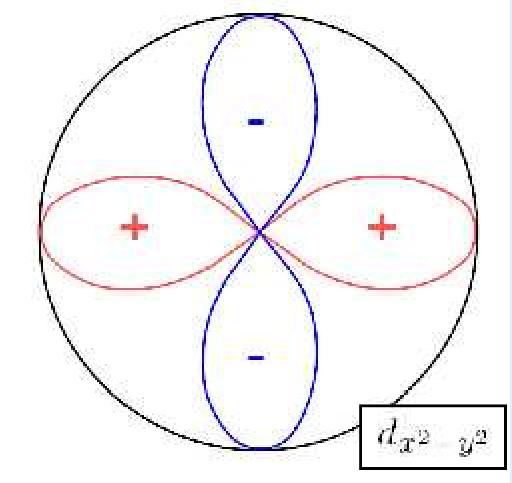
BCS High-Tc's

spin triplet: $\left(\chi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \ldots)\right)$

$\Rightarrow S = 1 \rightarrow$ space part must be odd.

$\Rightarrow$ p-wave (l = 1), f-wave (l = 3), etc...

Gap function: $\Delta(\mathbf{k})$ has a lower symmetry than the Fermi surface



As the gap disappears along some directions of the Fermi surface ("nodes"), extremely-low-energy quasiparticles excitations (and therefore significant pair-breaking) may occur at very low temperature

s-wave Superconducting Gap

- BCS conventional pairing:
  isotropic *s*-wave pairing

From μSR:

$$\sigma_\mu \propto \frac{1}{\lambda^2} = \frac{\mu_0 e^2}{m} n_S$$

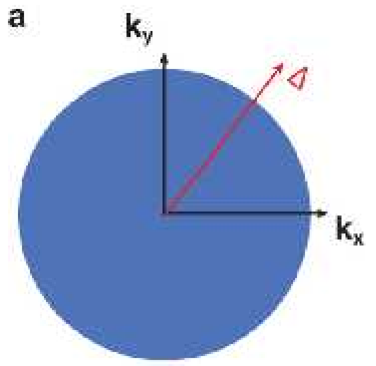
$$n_S(T) = n_S(0)\left(1 - \frac{2}{k_B T}\int_0^\infty f(\varepsilon, T)[1 - f(\varepsilon, T)]\, d\varepsilon\right)$$

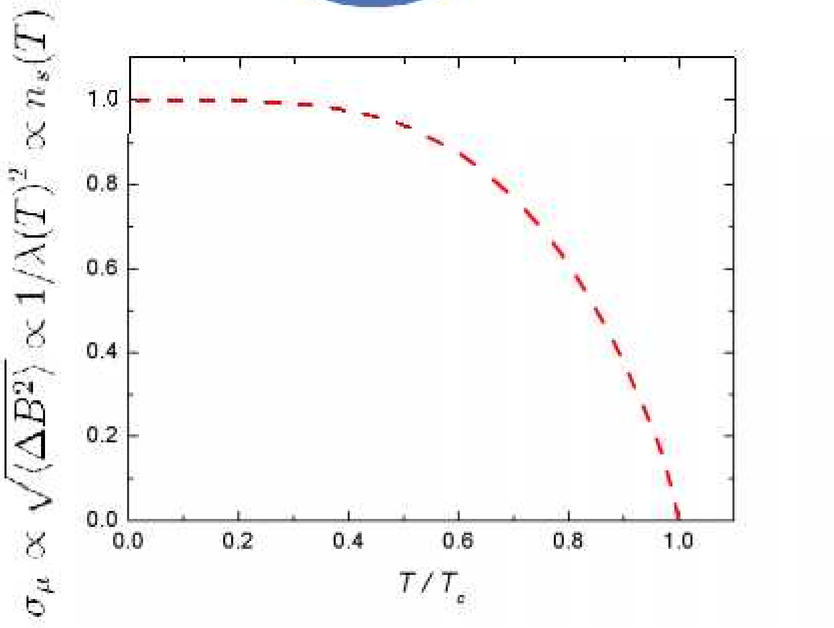
and for an isotropic energy gap (s-wave):

$$n_S(T) = n_S(0)\left(1 - \sqrt{\frac{2\pi\Delta(0)}{k_B T}} \exp\left[-\frac{\Delta(0)}{k_B T}\right]\right)$$

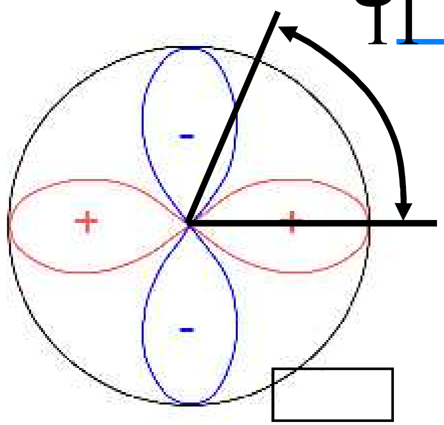
B. Mühlschlegel, Z. Phys. 155, 313 (1959)

s-wave gap





d-wave Pairing Symmetry



d-wave SC state:

$$n_s(T) = n_s(0)\left(1 - \frac{1}{\pi k_B T}\int_0^{2\pi}\int_0^{\infty} f(\epsilon, T)\,[1 - f(\epsilon, T)]\, d\varphi\, d\epsilon\right)$$

with:

$$f(\epsilon, T) = \left(1 + \exp\left[\sqrt{\epsilon^2 + [\Delta_s(T)\cos(2\varphi)]^2}/k_B T\right]\right)^{-1}$$

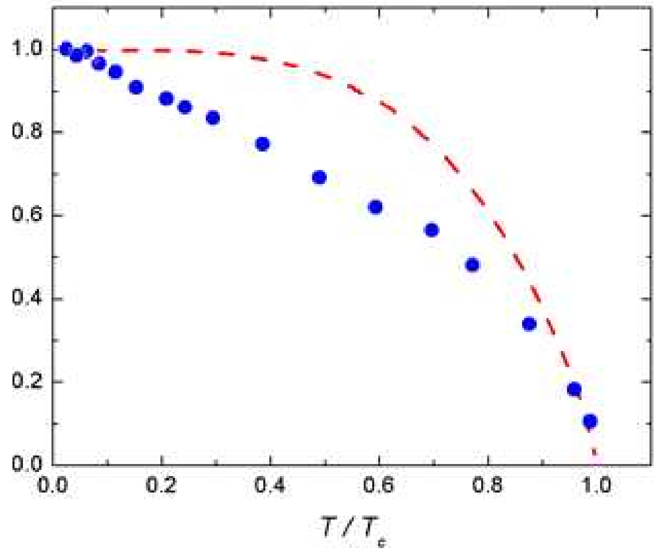
remembering that:

$$\lambda = \sqrt{\frac{m}{\mu_0 e^2 n_s}}$$

one gets: (for $T \ll T_c$)

$$n_s(T) \propto n_s(0)\left(1 - 2C\frac{T}{\Delta_s(0)}\right)$$

$$\lambda(T) \propto \lambda(0)\left(1 + C\frac{T}{\Delta_s(0)}\right)$$

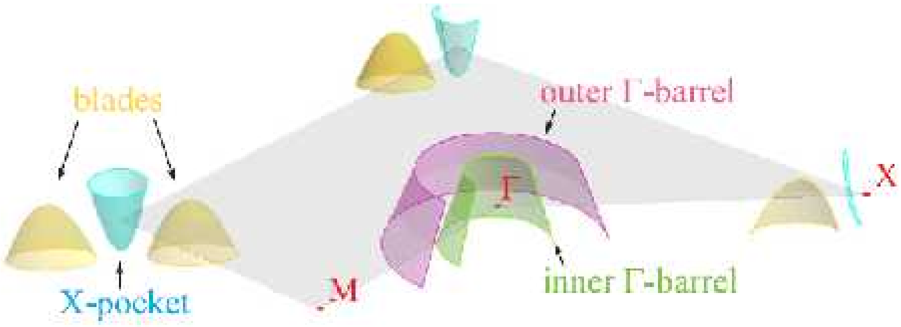


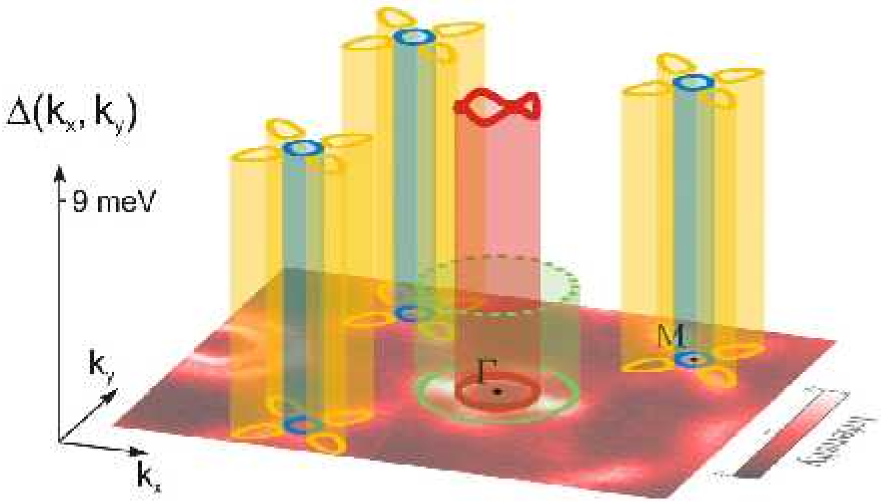
Single crystal $YBa_2Cu_3O_{6.95}$

*J.E. Sonier et al, PRL **72**, 744 (1994)*

Multiband Superconductivity

ARPES on $Ba_{1-x}K_xFe_2As_2$:

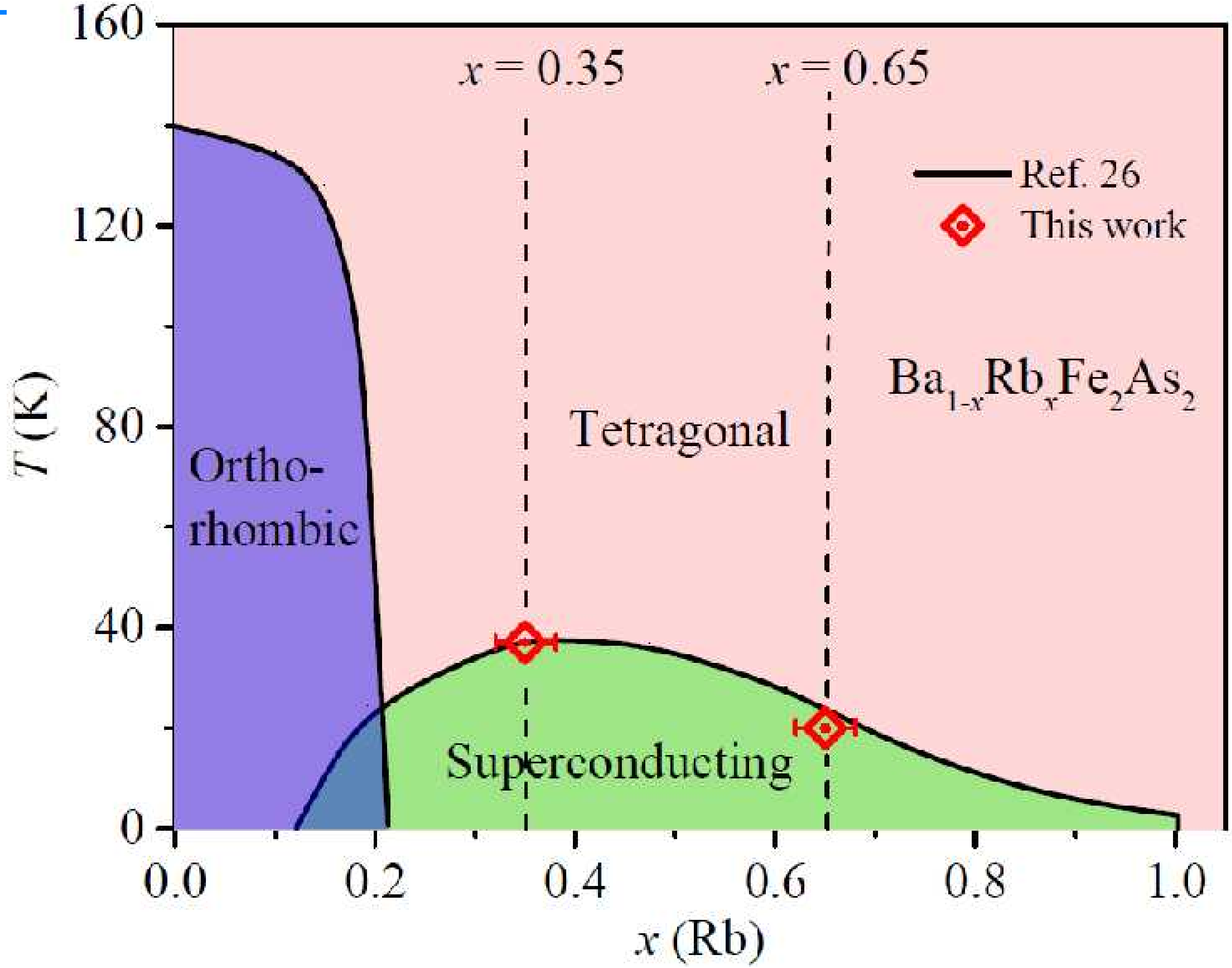


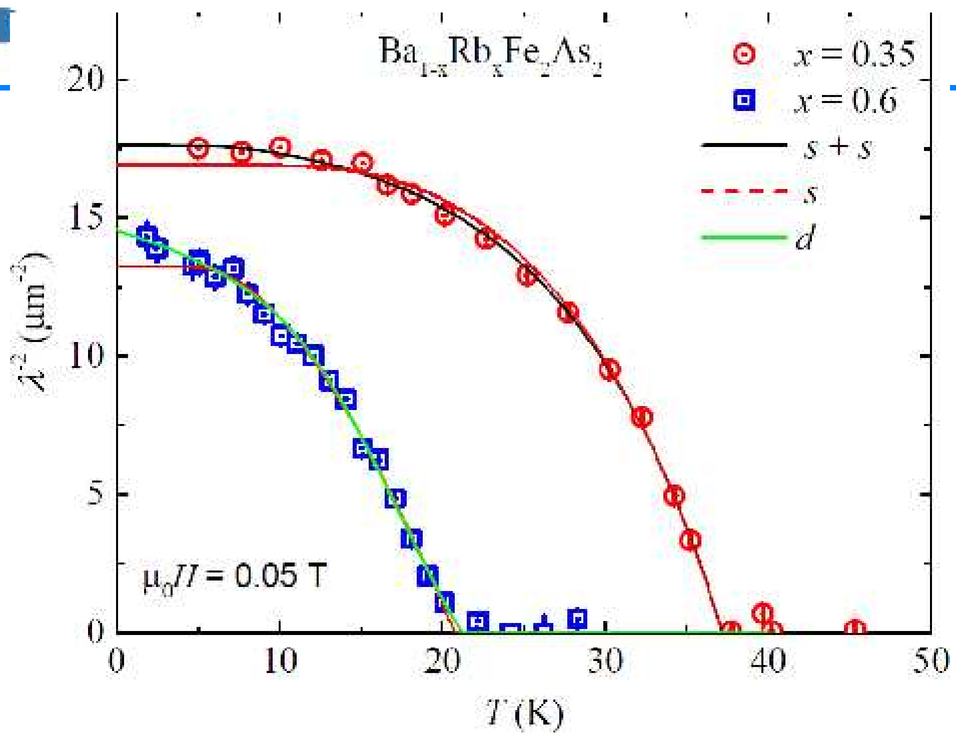


V. B. Zabolotnyy *et al.*, Nature **457**, 569 (2009).
D. V. Evtushinsky *et al.*, Phys. Rev. B **79**, 054517 (2009).
D. V. Evtushinsky *et al.*, New J. Phys. **11**, 055069 (2009).

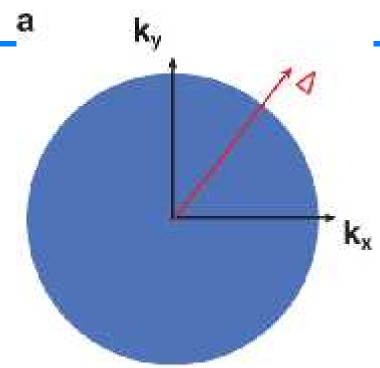
- **Multiband superconductivity**
- **Nodeless gaps**

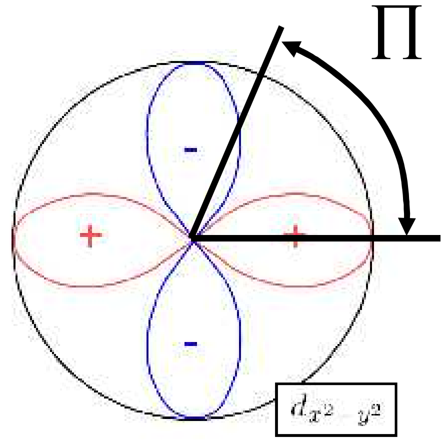
$$\frac{\lambda^{-2}(T)}{\lambda^{-2}(0)} = \omega_1 \frac{\lambda^{-2}(T,\Delta_{0,1})}{\lambda^{-2}(0,\Delta_{0,1})} + \omega_2 \frac{\lambda^{-2}(T,\Delta_{0,2})}{\lambda^{-2}(0,\Delta_{0,2})}$$


160
120
80
40
0
T (K)
0.0
0.2
0.4
0.6
0.8
1.0
x (Rb)
x = 0.35
x = 0.65
Ref. 26
This work
$Ba_{1-x}Rb_xFe_2As_2$
Tetragonal
Ortho-
rhombic
Superconducting



s-wave gap





$$\boxed{\rho_{tot}(T) = w\rho_1(T) + (1-w)\rho_2(T)}$$

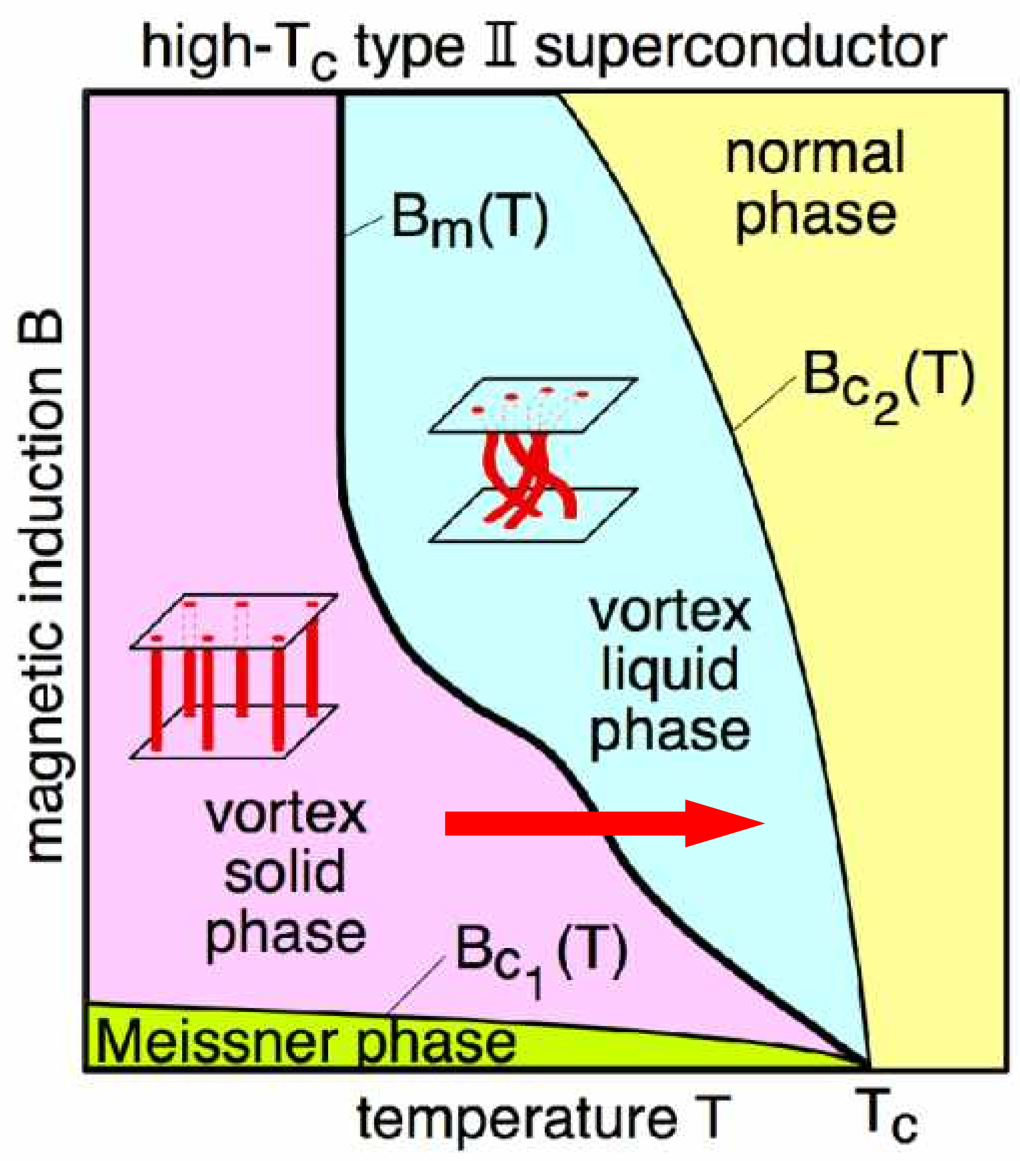
$$\rho_i(T) = \lambda_{ab}^{-2}(T, \Delta_{0,i})/\lambda_{ab}^{-2}(0, \Delta_{0,i}),\ i = 1, 2$$

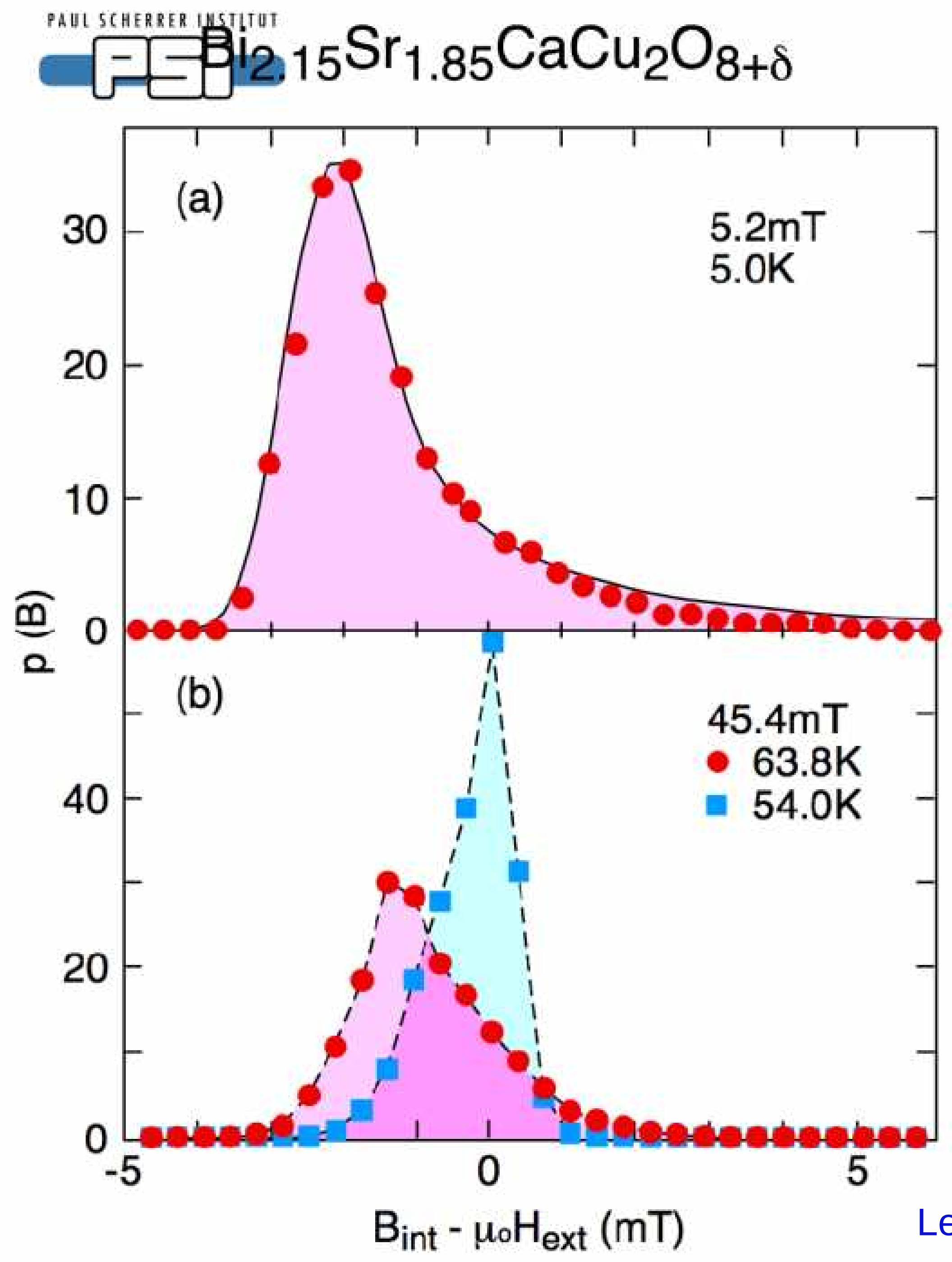
$$\Delta_{0,1} = 9.06(2)\mathrm{meV},\ \Delta_{0,2} = 1.50(2)\mathrm{meV},\ w = 0.51(2)$$

$\Delta_{0,1}$=1.1(3) meV, $\Delta_{0,2}$=7.5(2) meV, ω = 0.15(3).

Guguchia *et al.*, Phys. Rev. B **84,** 094513 (2011).

Melting of the vortex lattice



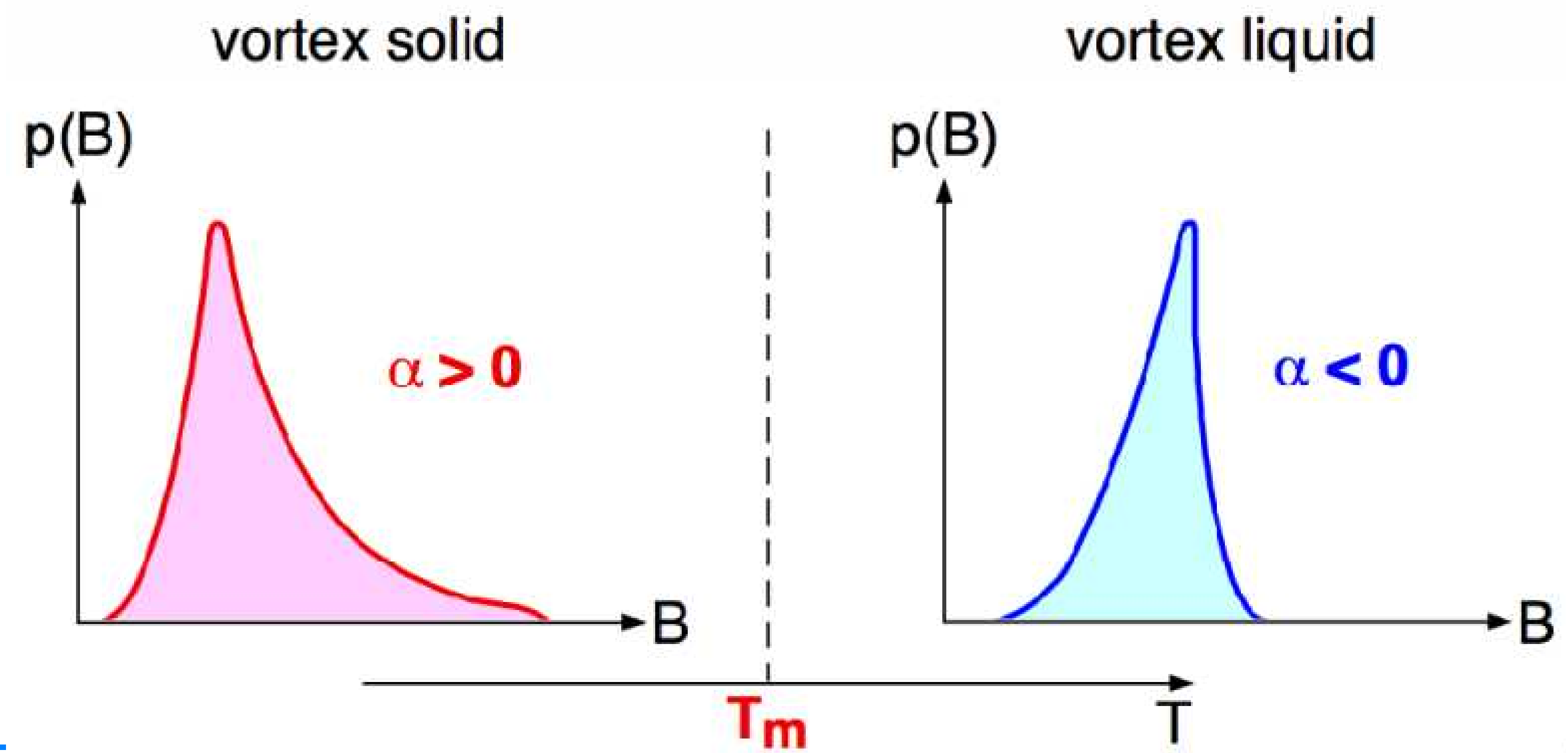


Vortex lattice melting

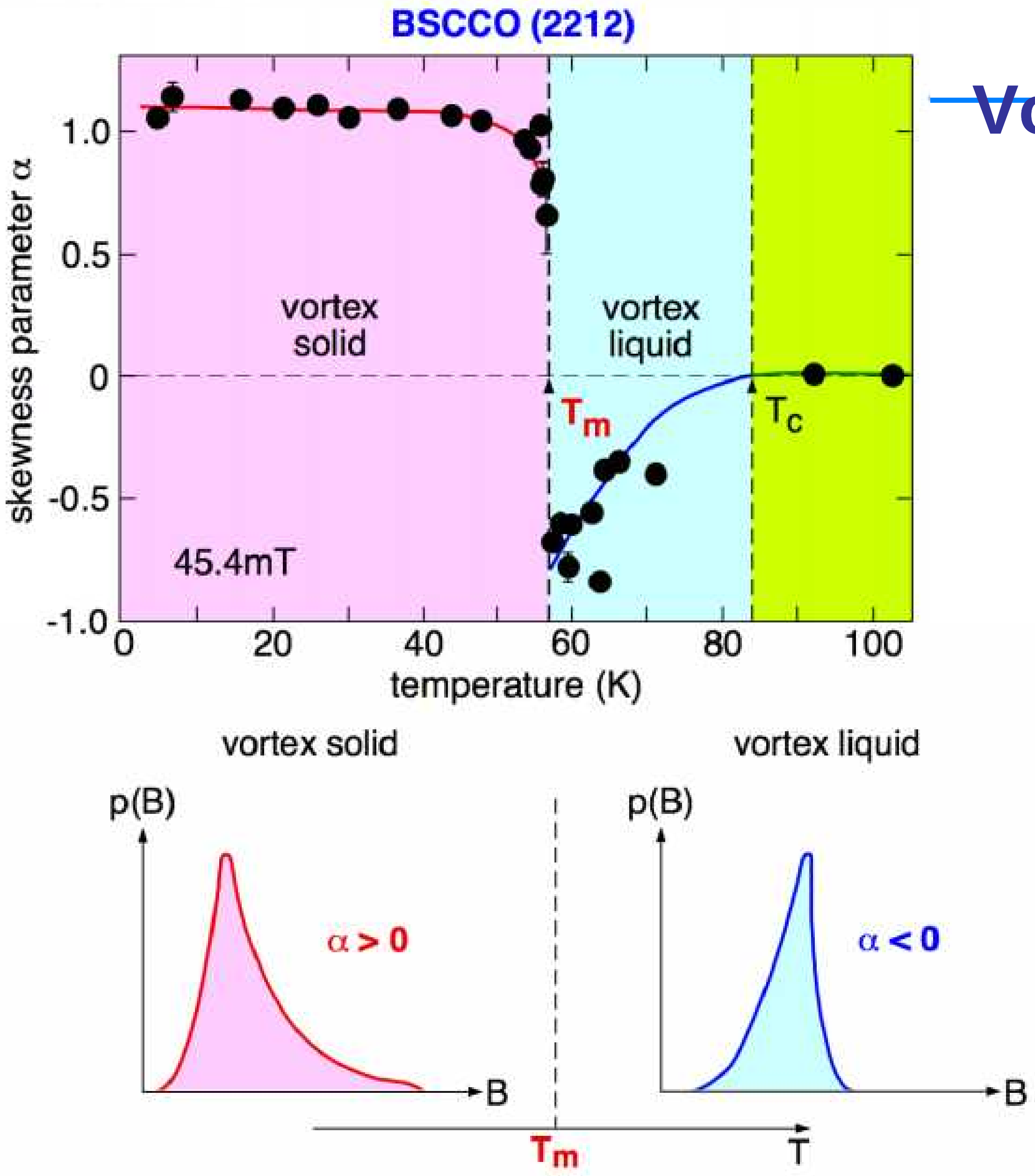
Lee *et al.*, Phys. Rev. Lett. **71**, 3862 (1993)

Lineshape asymmetry parameter α
“skewness parameter”

$$\alpha = \langle(\Delta B)^3\rangle^{1/3} / \langle(\Delta B)^2\rangle^{1/2}$$



Vortex lattice melting

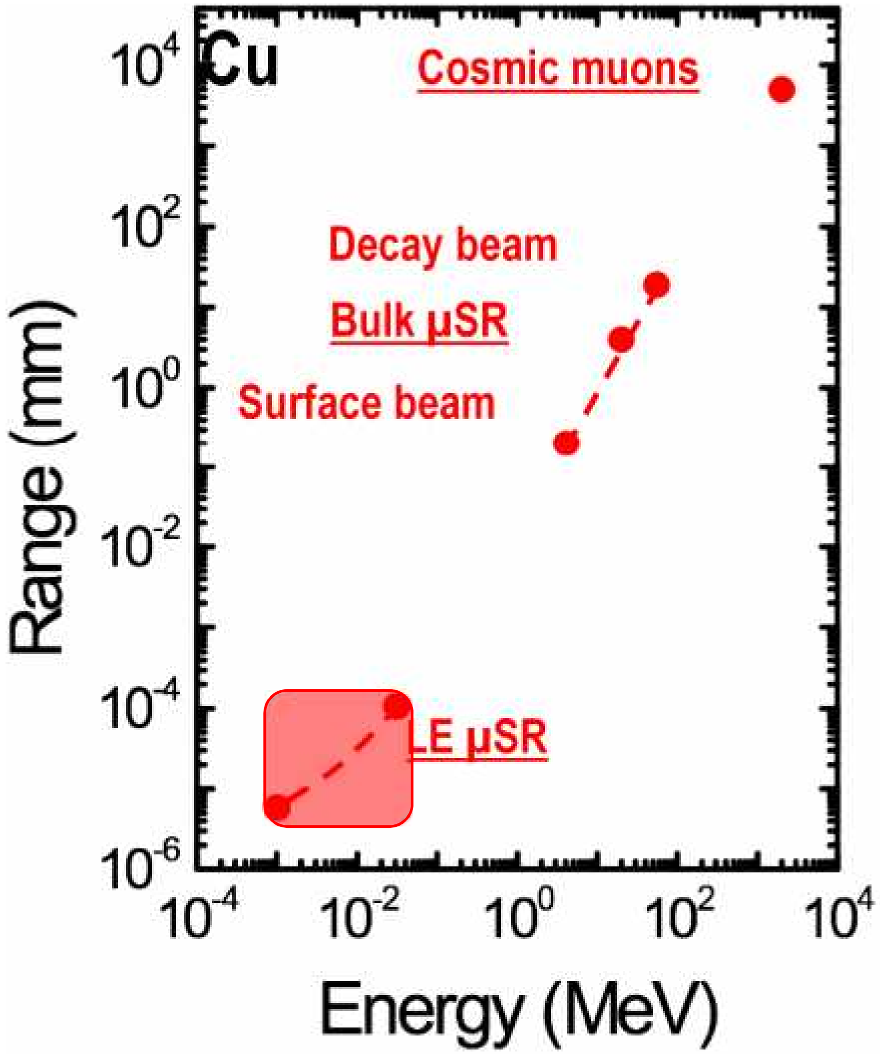


$$\alpha = \langle(\Delta B)^3\rangle^{1/3} / \langle(\Delta B)^2\rangle^{1/2}$$

Lee *et al.*, Phys. Rev. Lett. **71**, 3862 (1993)

Using Low Energy Muon Spin Rotation to Study Superconducting Materials

Low Energy μSR



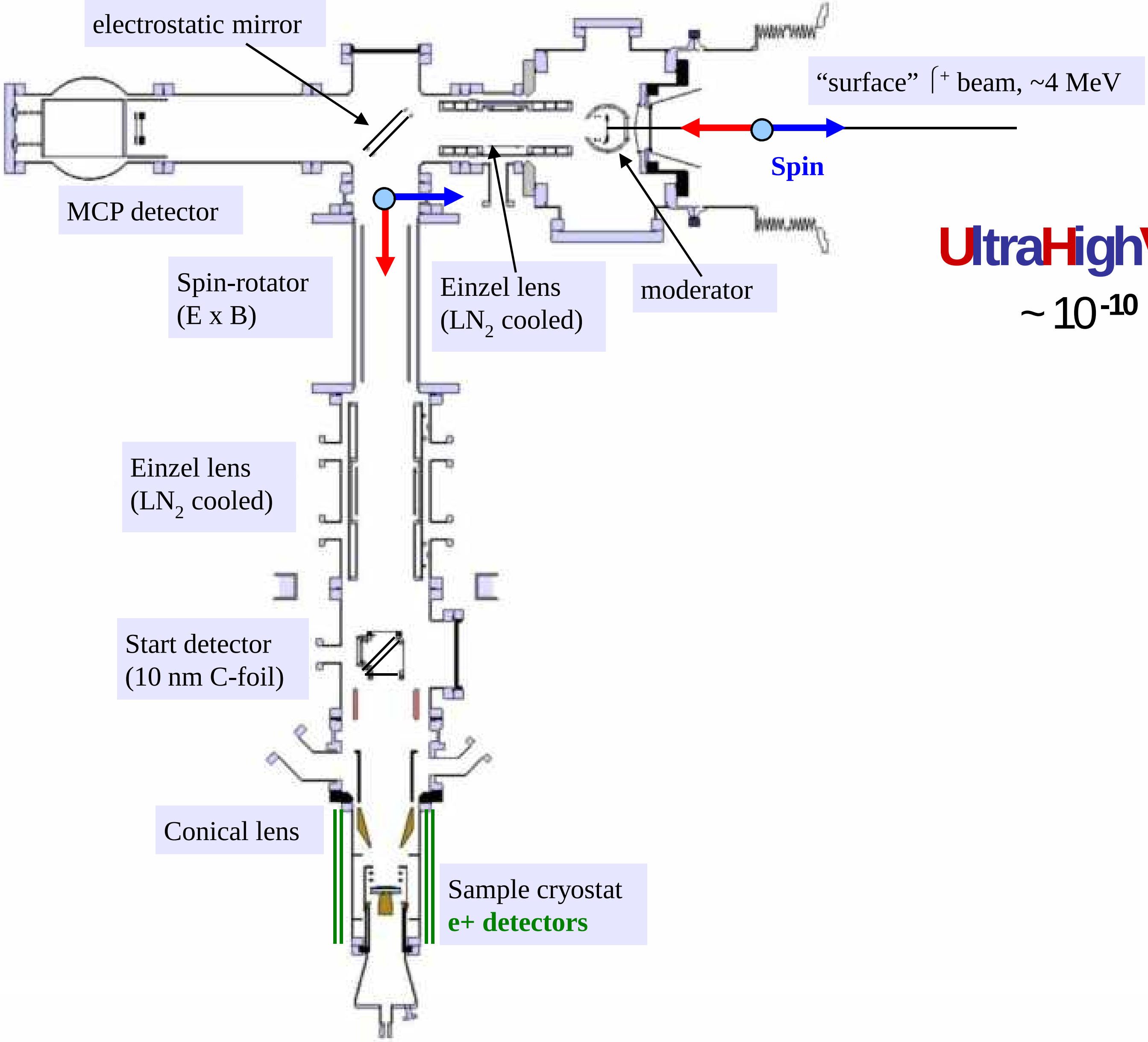
Variable muon energy: 0 – 30 keV

▼

Depth-sensitive local magnetic spin probe on the nm length scale

- Thin films
- Near-surface regions
- Multilayers
- Buried interfaces

Low Energy μSR Apparatus



UltraHighVacuum

~ 10^{-10} mbar

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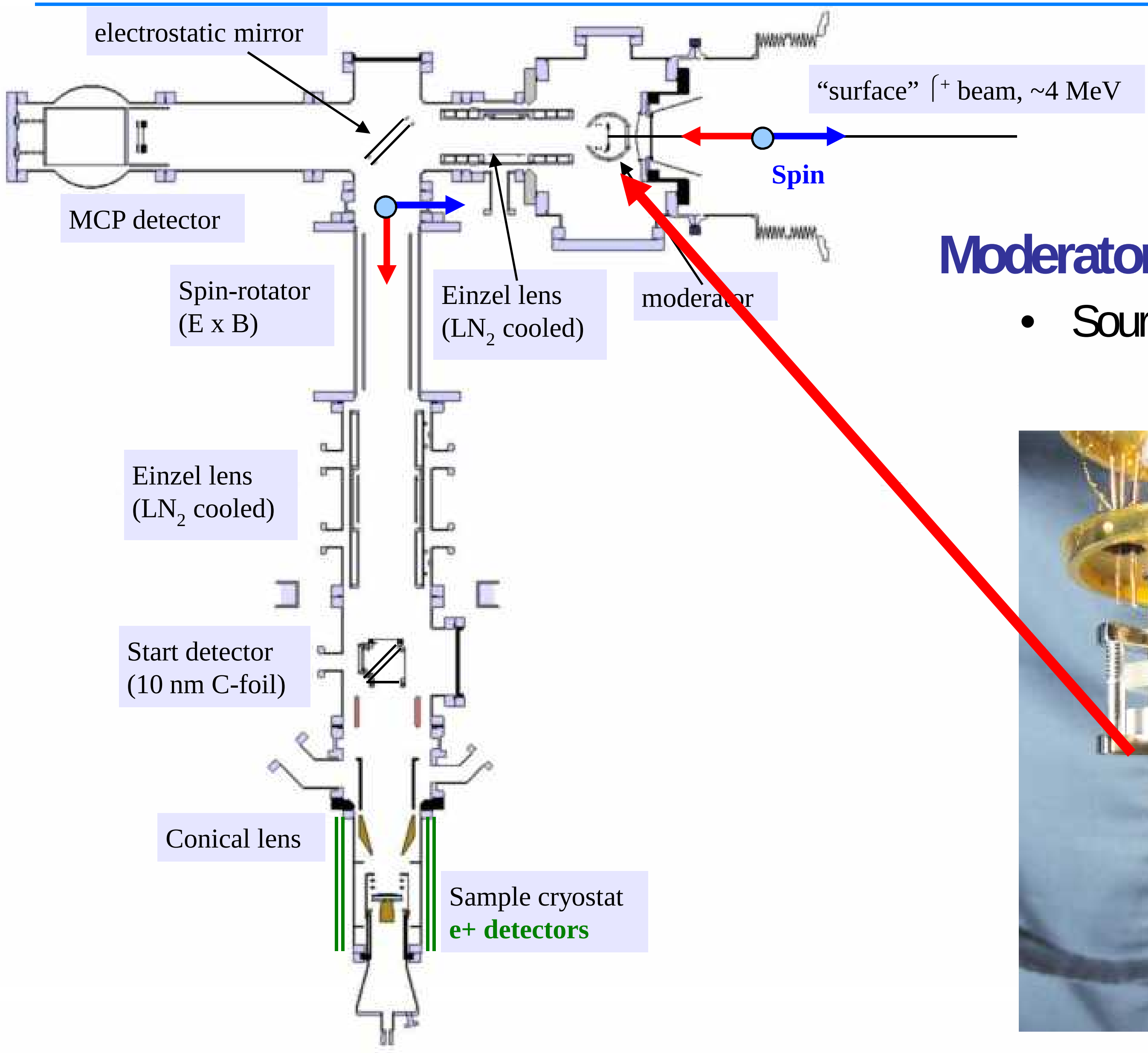
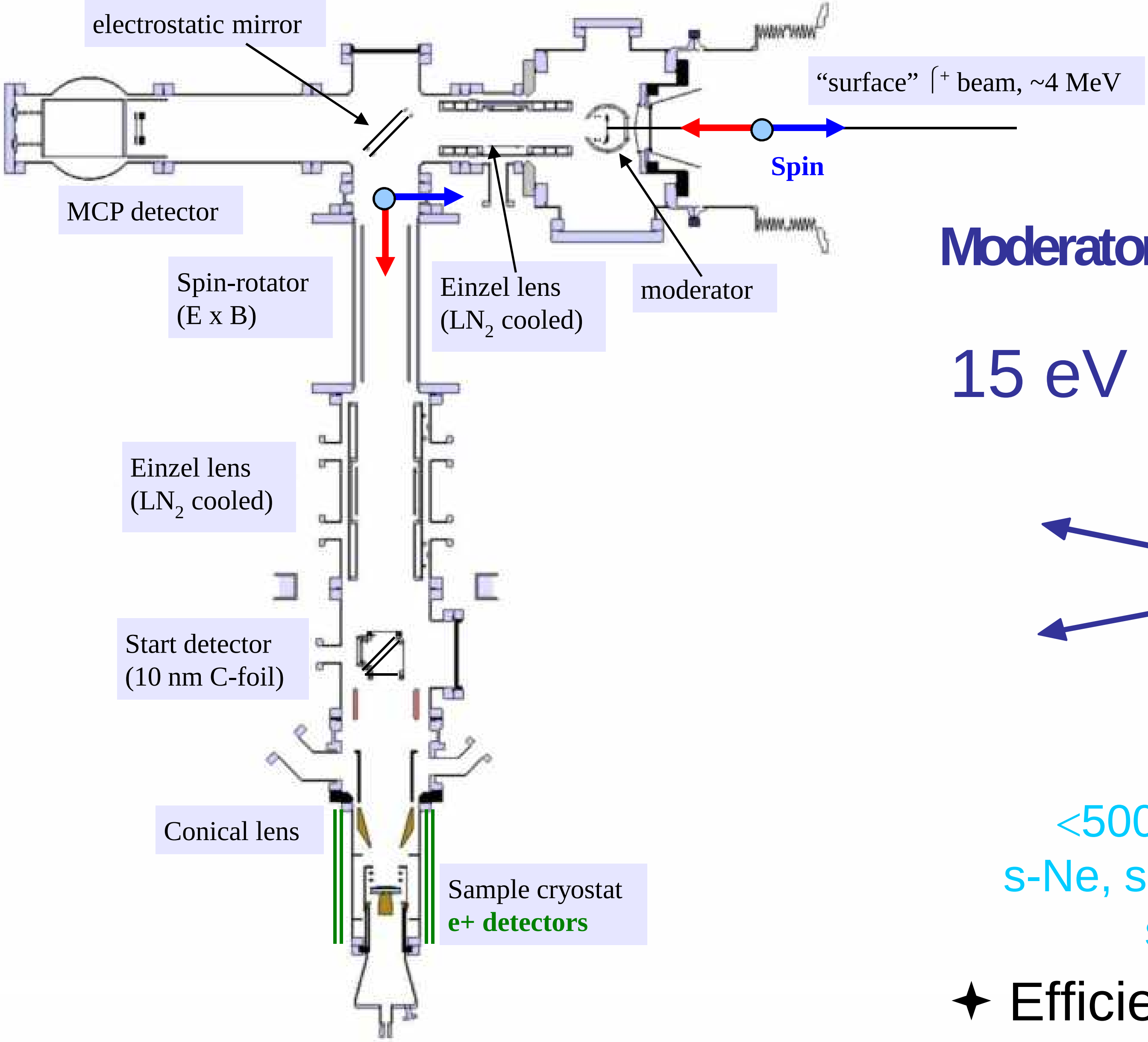
Low Energy μSR Apparatus



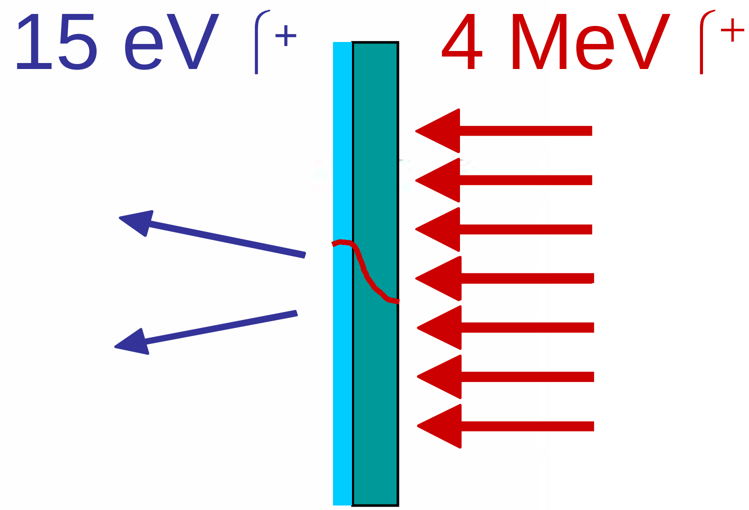
Moderator

- Source of low energy muons

Low Energy μSR Apparatus



Moderator



<500 nm <100 μm

s-Ne, s- Ar, Ag at 6K

s-N_2

✦ Efficiency ~ 10^{-4} – 10^{-5}

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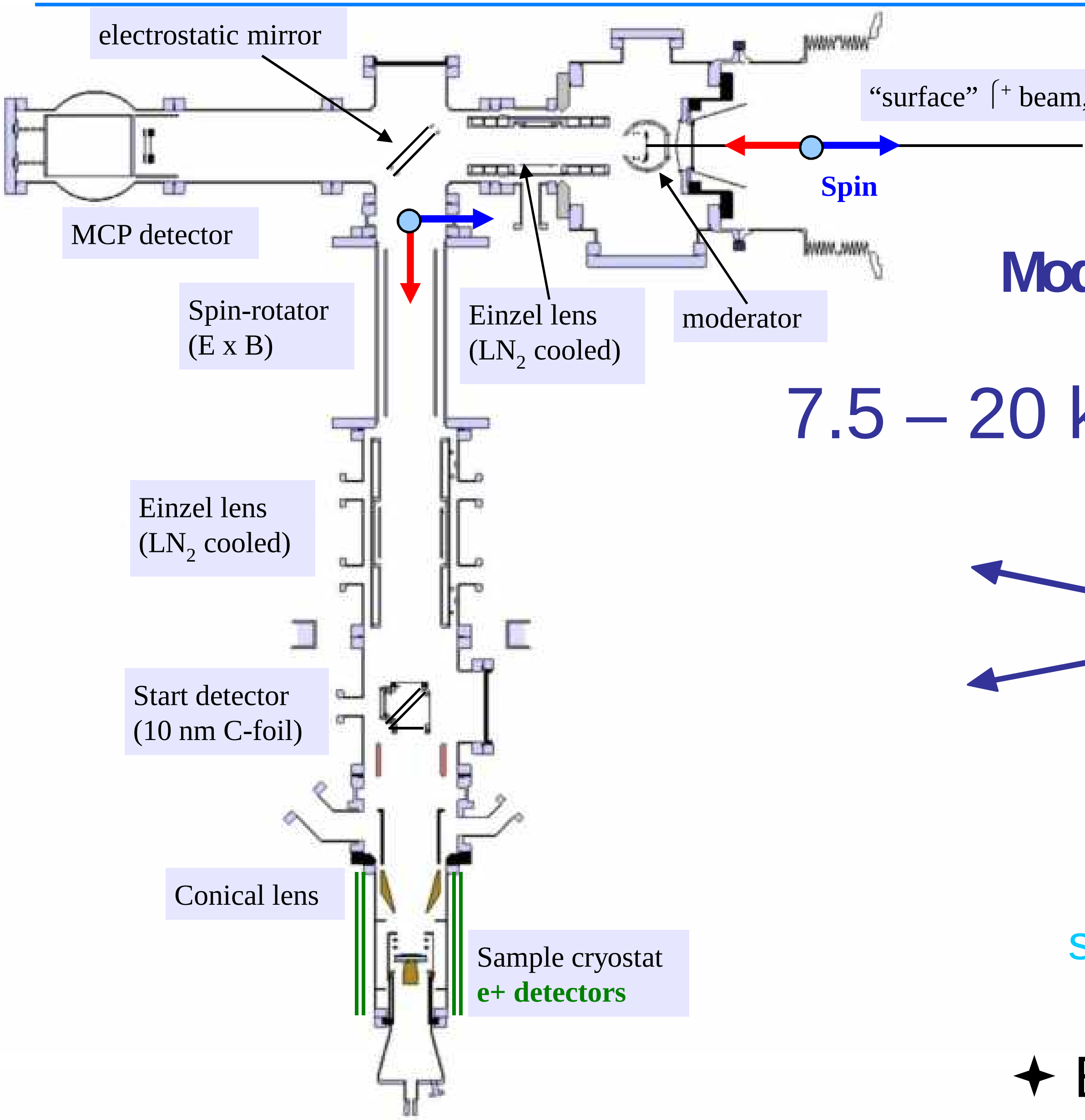
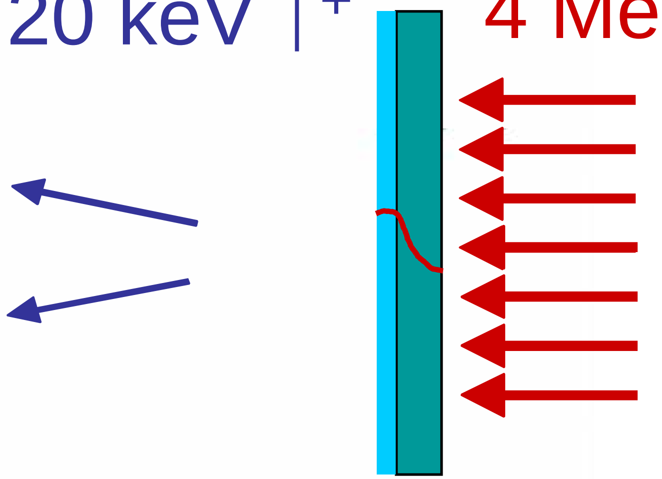
Low Energy μSR Apparatus



Moderator

7.5 – 20 keV μ^+

4 MeV μ^+



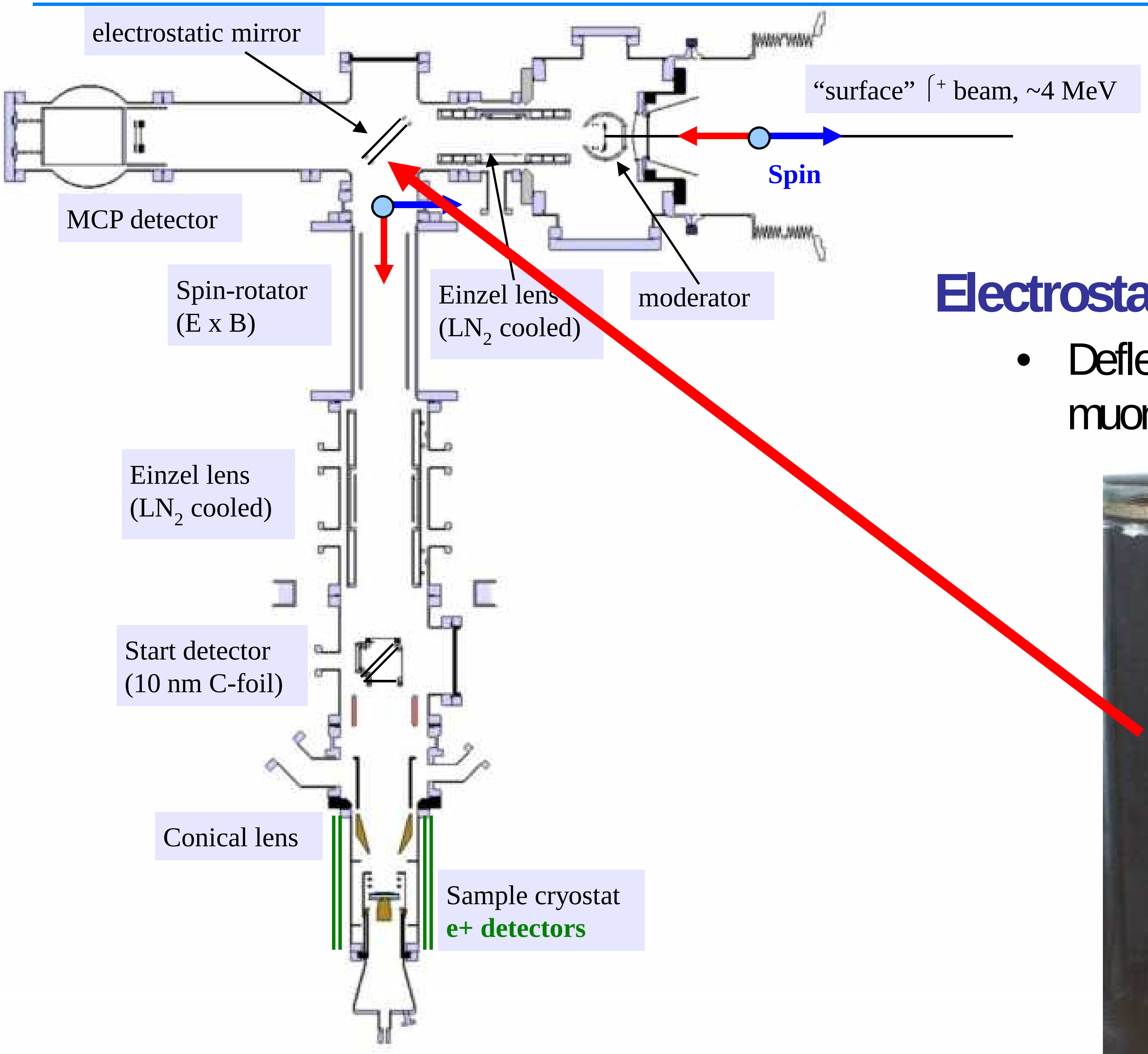
<500 nm <100 μm

s-Ne, s- Ar, Ag at 6K

s-N_2

✦ Efficiency ~ 10^{-4} – 10^{-5}

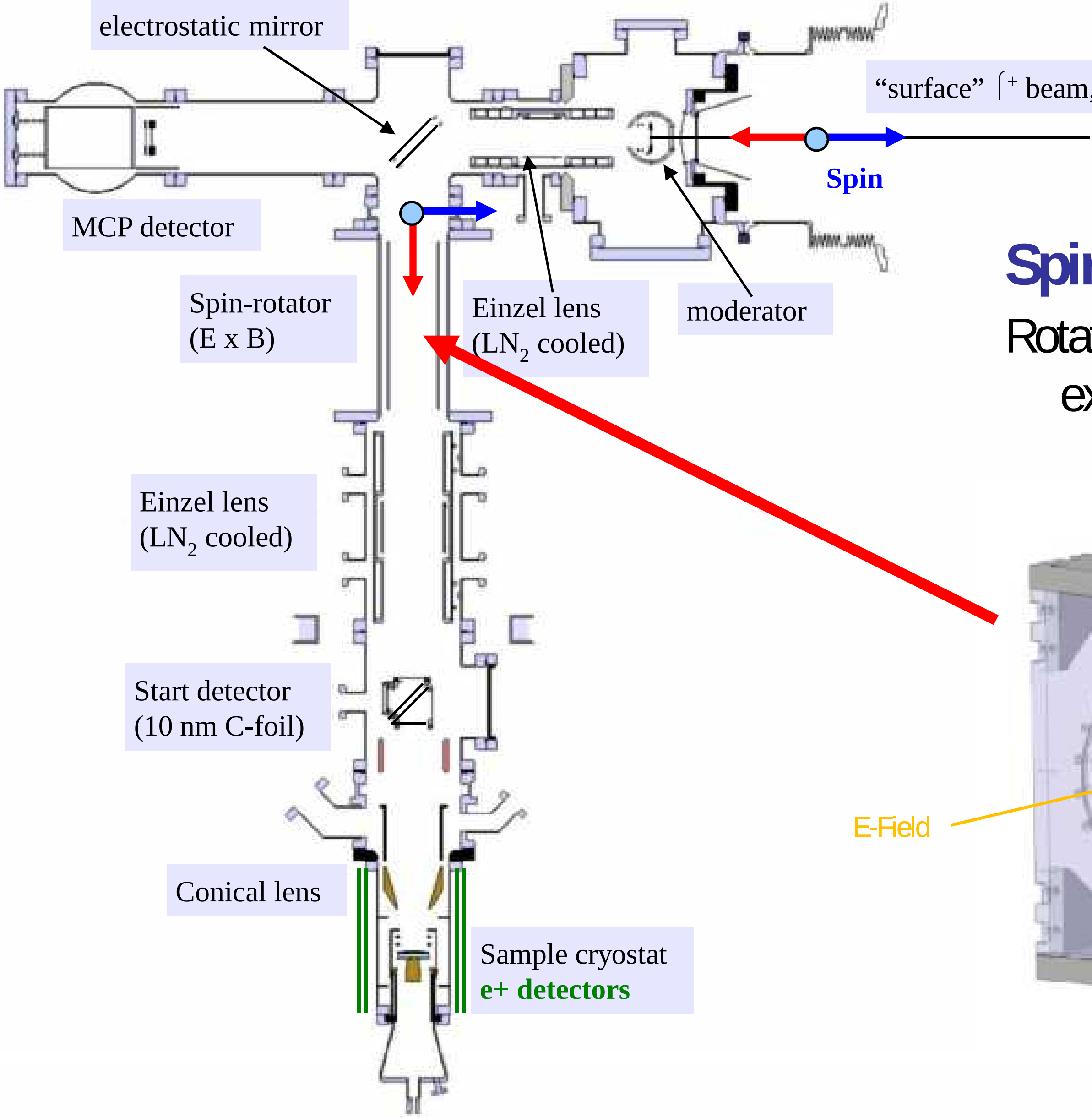
Low Energy μSR Apparatus



Electrostatic mirror

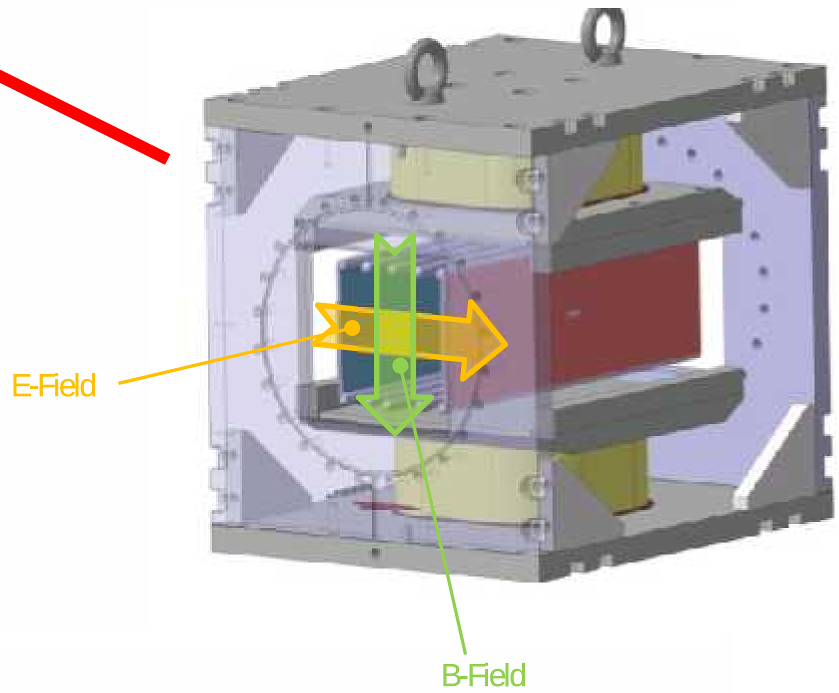
- Deflection of the low energy muons

Low Energy μSR Apparatus

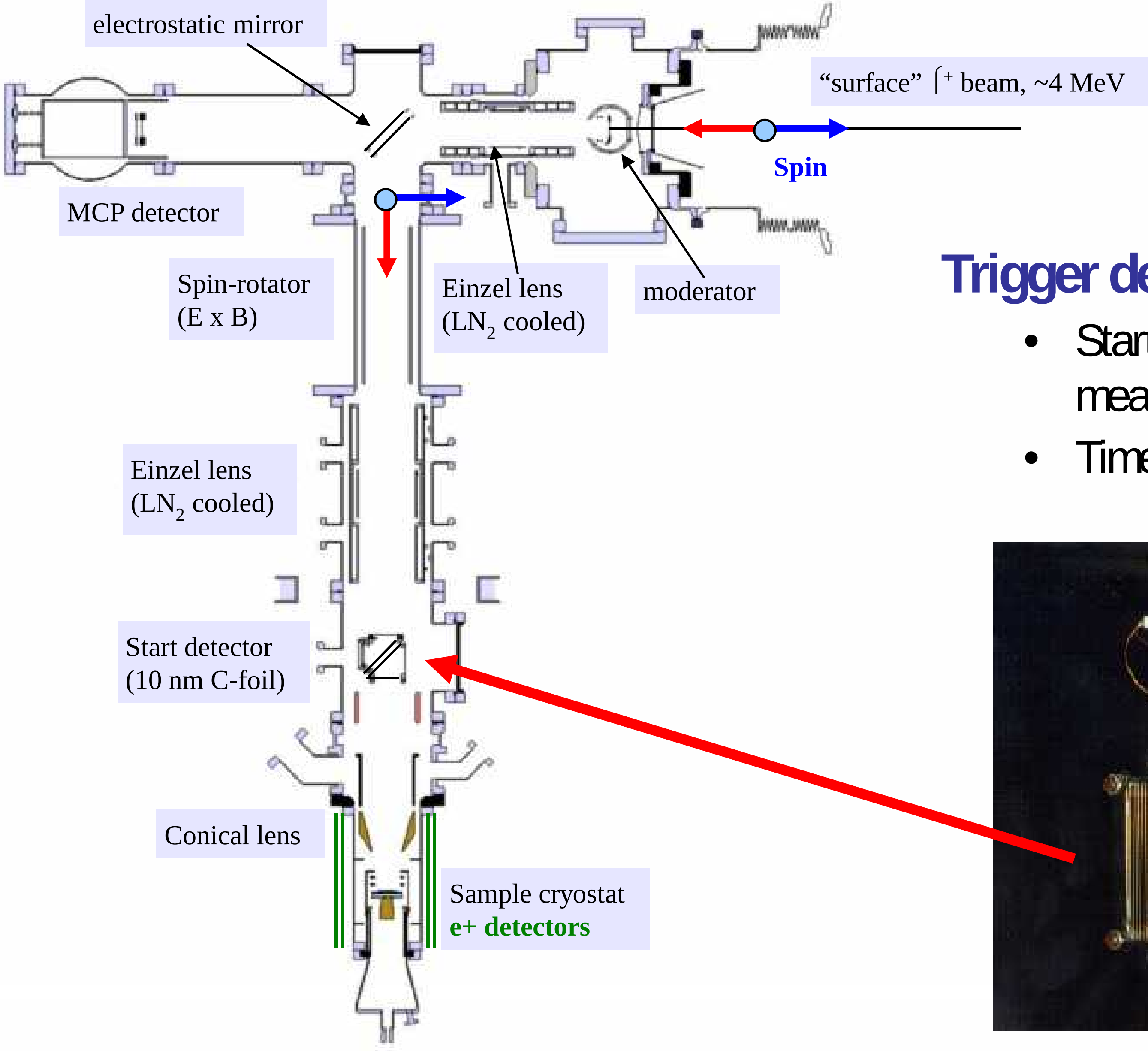


Spin Rotator

Rotates muon spin for different experimental conditions

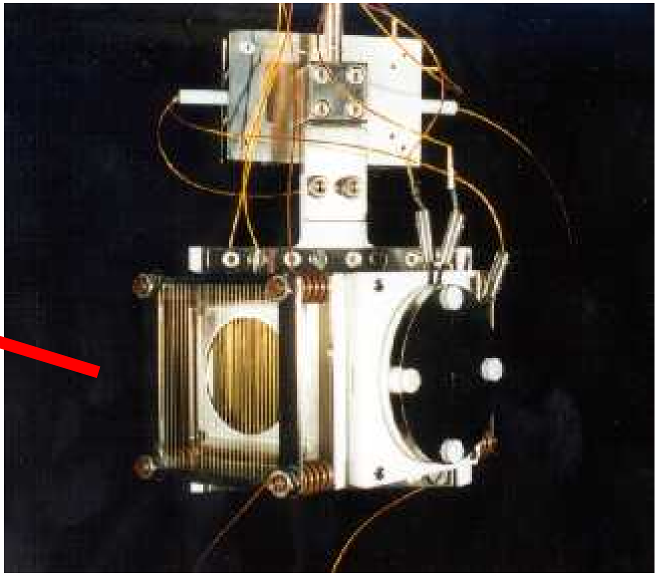


Low Energy μSR Apparatus



Trigger detector

- Start of the LE-μSR measurement
- Time-of-flight measurements



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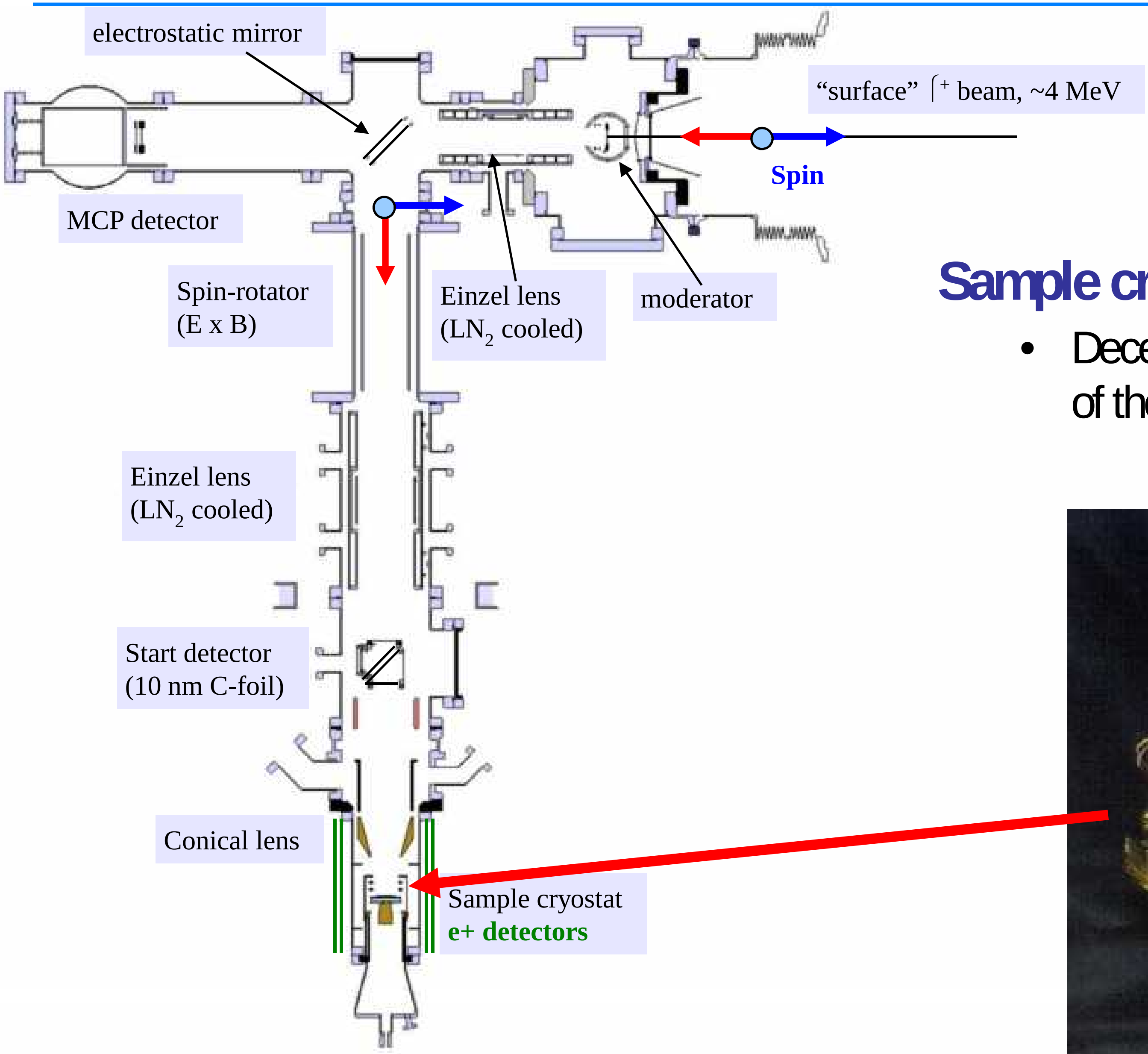
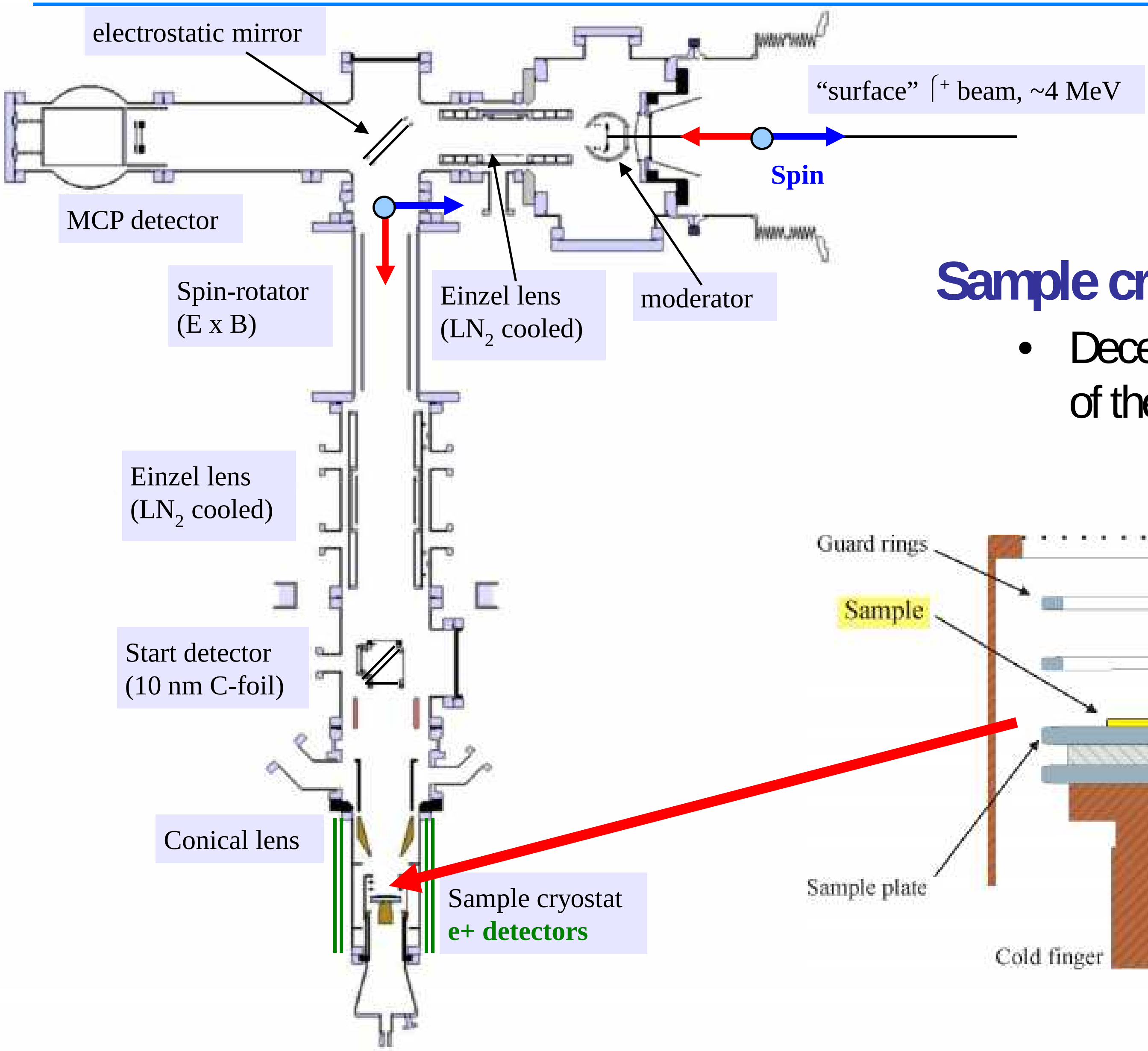
Low Energy μSR Apparatus



Sample cryostat

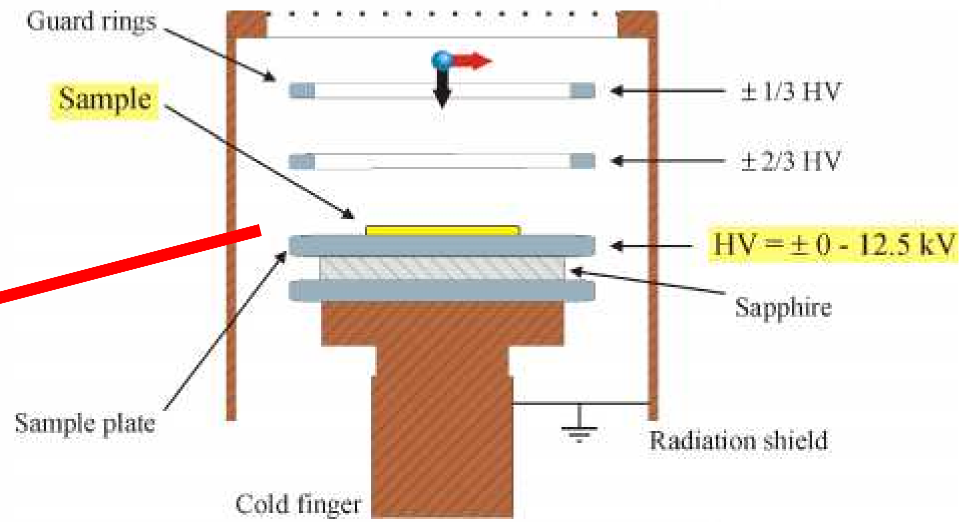
- Deceleration and acceleration of the μ^+

Low Energy ⌈SR Apparatus



Sample cryostat

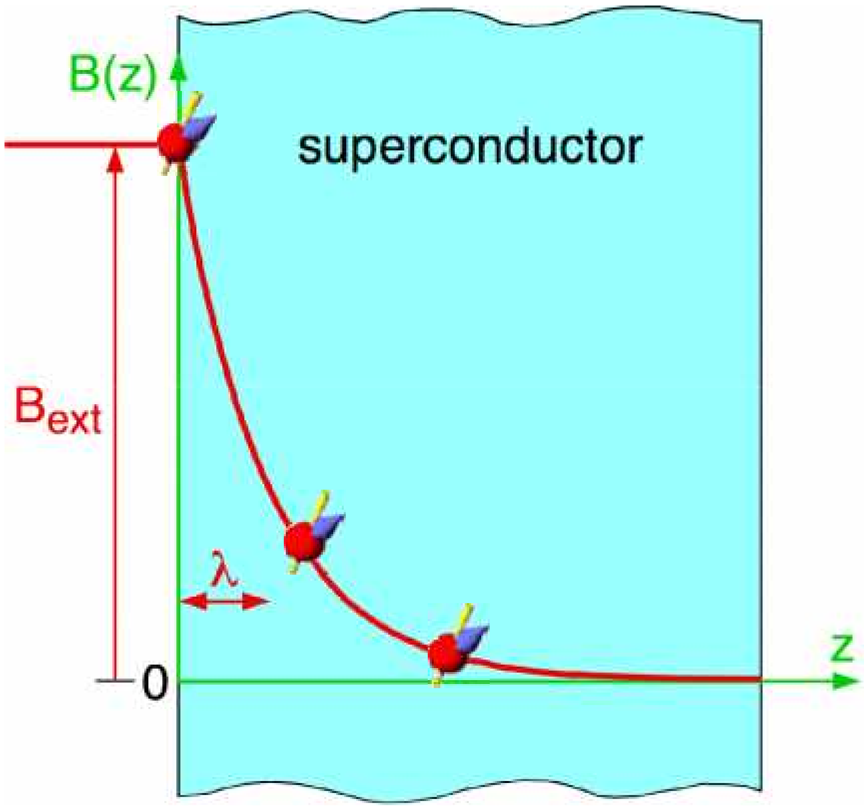
- Deceleration and acceleration of the ⌈+



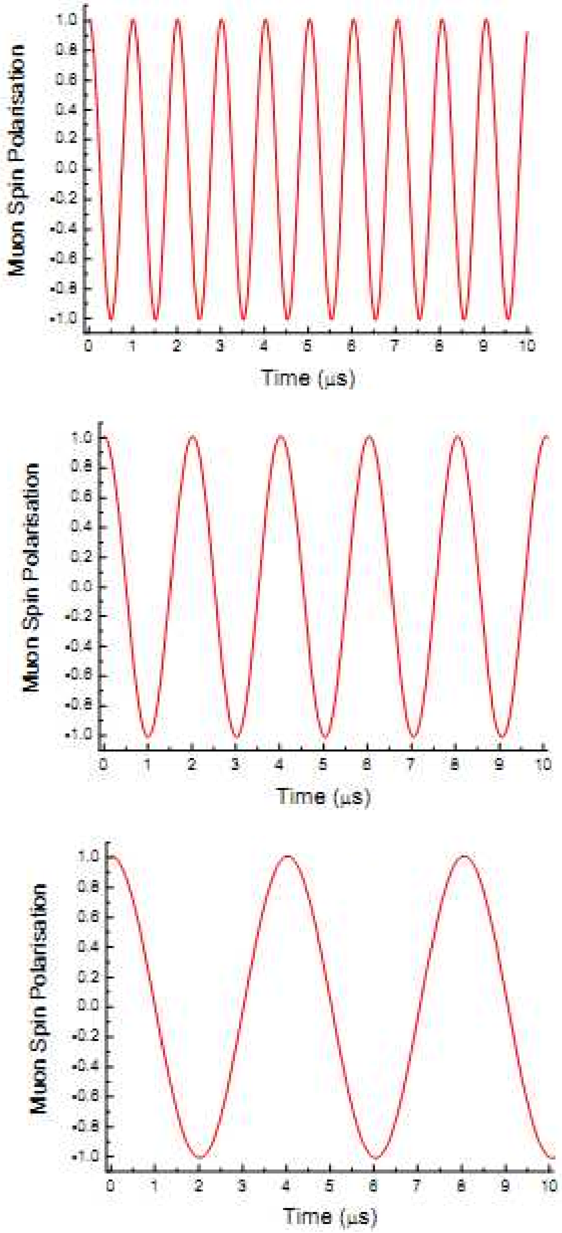
Example:
Magnetic field profile in type-I and type-II superconductors

Depth dependent μSR measurements

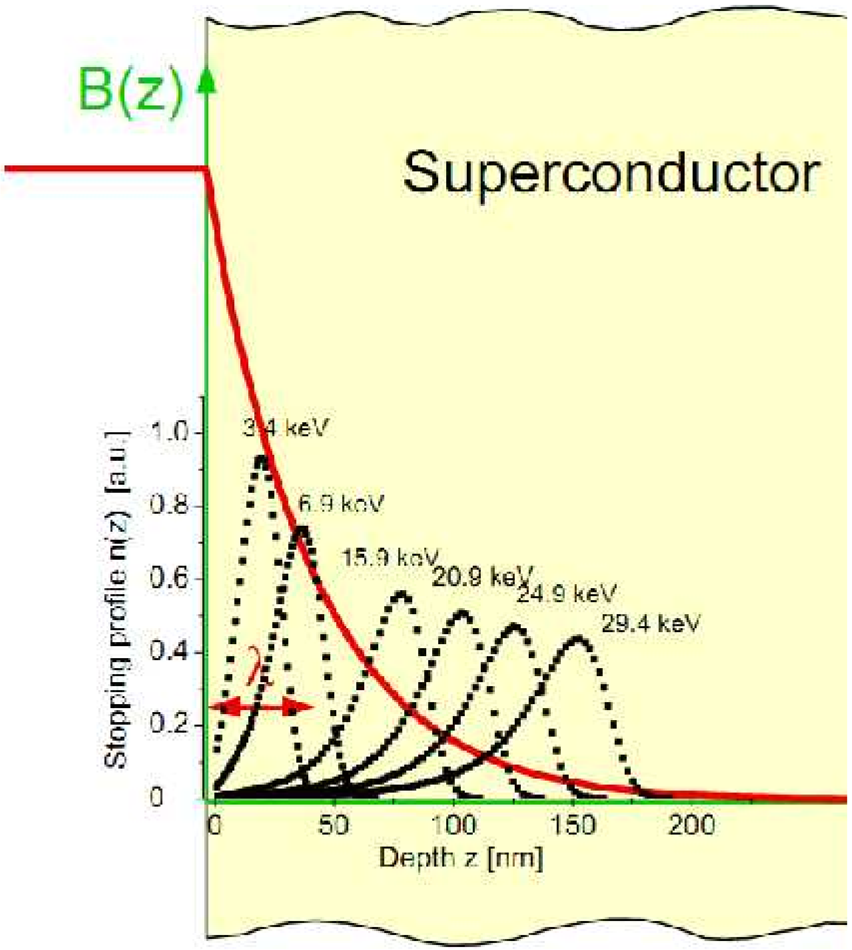
$$B(z) = B_{ext}\exp(-z/\lambda)$$



$$\omega_\mu(z) = \gamma_\mu B_{loc}(z)$$



Jackson *et al*., Phys. Rev. Lett. **84**, 4958 (2000)



→ Magnetic field profile B(z) over nm scale

n(z,E): muon implantation profile for a particular muon energy E

μSR experiment ⇒ magnetic field probability distribution p(B, E) sensed by the muons

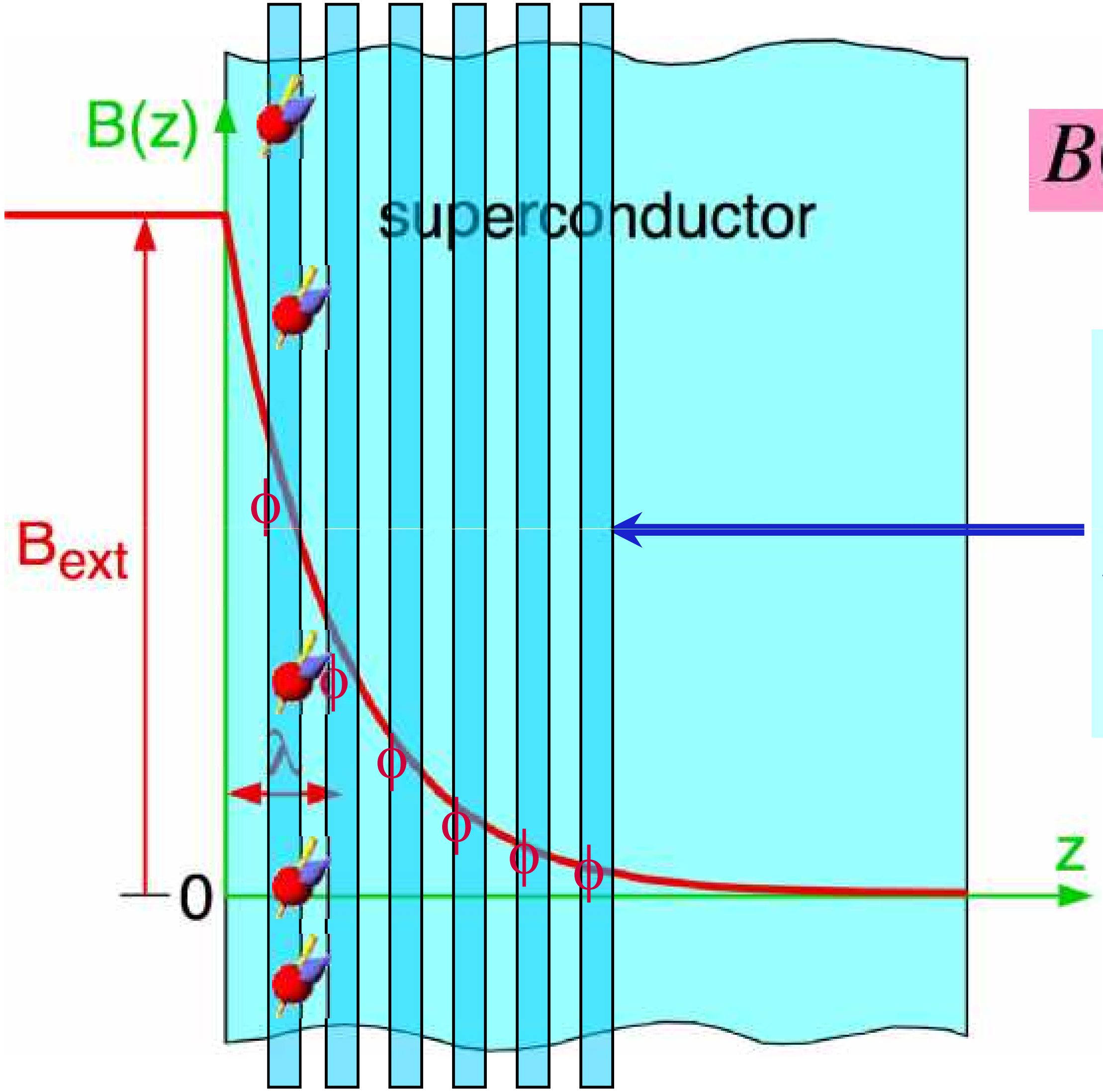
$$n(z,E)\,\mathrm{d}z = p(B,E)\,\mathrm{d}B$$

$$\int_0^z n(\zeta,E)\,\mathrm{d}\zeta = \int_{B(z)}^{\infty} p(\beta,E)\,\mathrm{d}\beta$$

$\Rightarrow$ B(z)

Jackson *et al*., Phys. Rev. Lett. **84**, 4958 (2000)

Direct measurement of the magnetic penetration depth



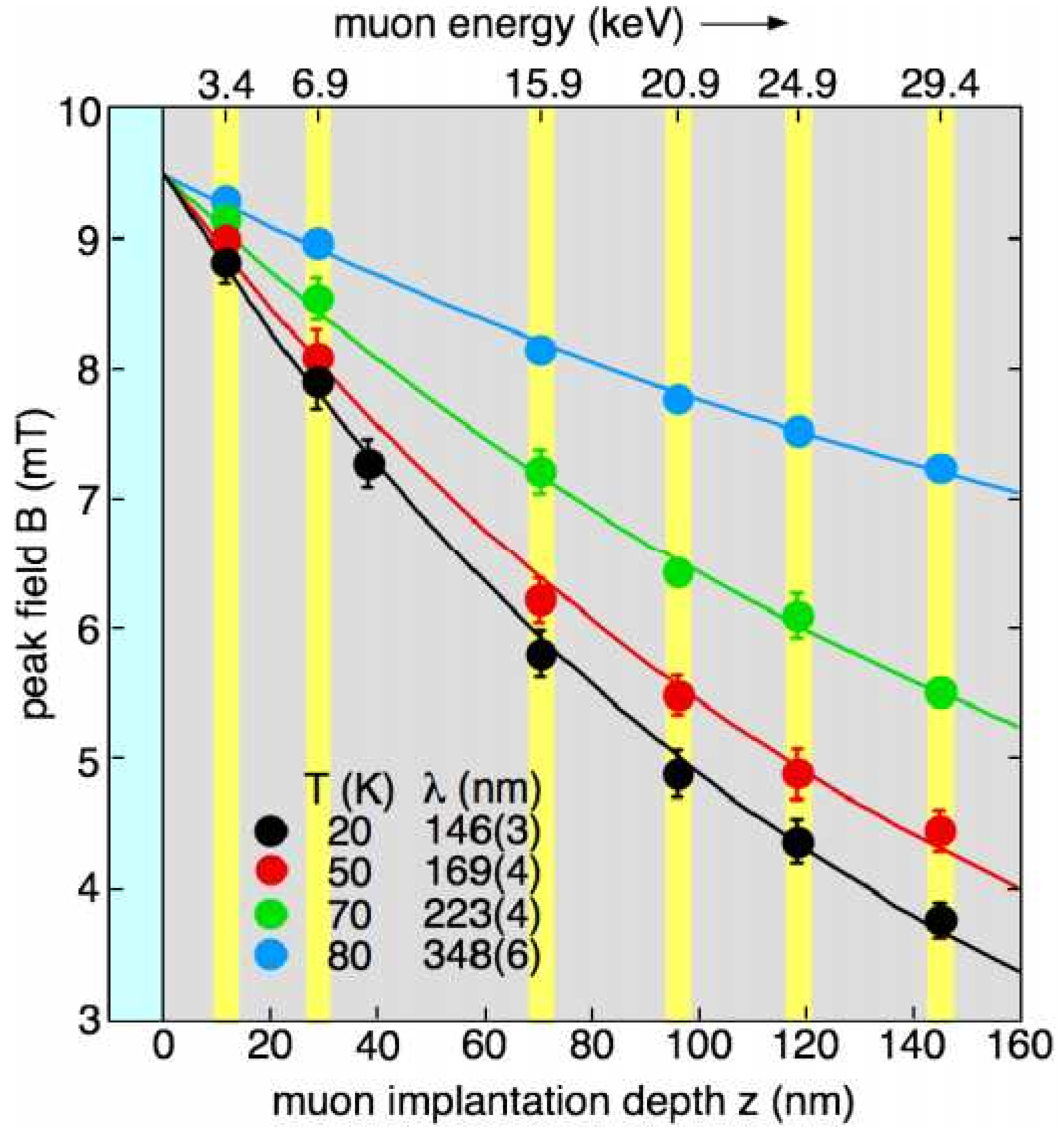
$$B(z) = B_{ext}\exp(-z/\lambda)$$

⌠SR experiment ®
magnetic field probability distribution p(B) at the muon site

® p(B) ® B(z)

Jackson *et al*., Phys. Rev. Lett. **84**, 4958 (2000)

Direct measurement of λ in a $YBa_2Cu_3O_{7-δ}$ film



Jackson *et al.*, Phys. Rev. Lett. **84**, 4958 (2000)

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Summary

Muon Spin Rotation / Relaxation (μSR)

Magnetism:

- **Local probe**
  - Magnetic volume fraction
- **μSR frequency**
  - Magnetic order parameter ($10^{-3} – 10^{-4}\ \mu_B$)
  - Temperature dependence
- **μSR relaxation rate**
  - Homogeneity of magnetism
- **Magnetic fluctuations**
  - Time window: $10^5 – 10^9$ Hz

Superconductivity:

- **Field distribution of vortex lattice**
  - Penetration depth
  - Coherence length
  - Vortex dynamics
- **Absolute determination of penetration depth $\lambda(0)$**
- **Temperature dependency of**
  - Penetration depth, $\lambda^{-2} \propto \langle \Delta B^2 \rangle^{1/2} \propto \sigma$
  - Superfluid density $n_s/m^* \propto \langle \Delta B^2 \rangle \propto \sigma$
  - → Symmetry of the SC gap function

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